

First Year Courses

MT110	Analysis
MT111	Analysis
MT112	Algebra
MT114	Methods
MT115	Probability and Numerical Analysis or Computer Programming
MT116	Mechanics
MT118	Algebra

MT111 and MT118 are for joint honours students in Computer Science and Mathematics and in Mathematics and Physics.

July 1984.

MF110AnalysisMichaelmas Term.

1. Real numbers, inequalities including triangle and circle. Irrationality of $\sqrt{2}$, real numbers as Dedekind cuts or infinite decimals, etc. Intervals. Supremum, infimum, existence thereof. (6)
2. Limits of sequences (ϵ), sums of sequences etc. Inequalities, sandwich rule. Infinite limits of oscillation. Limit implies bounded, monotonic bounded implies limit. Subsequences, Bolzano-Weierstrass theorem. (8)
3. Infinite series (ϵ), linear combinations, n -th term tends to zero, bracketing in a series. Series of positive terms, comparison, ratio, condensation, limit comparison tests. Absolute convergence. Alternating series. Rearrangement. Multiplication of series. (12)
4. Complex numbers, definition, they form a field, $|z|$, z , triangle and circle inequalities. Polar representation (non-rigorous). Sequences and series. Power series, radius of convergence. (6)
5. Limits and continuity (δ) in real variable, sums, products, $f(g(x))$. Intermediate value theorem, inverse function theorem for monotonic continuous functions. Weierstrass theorem on infinite series of continuous functions. Definition of $\exp$ and $\log$, a^b , $\log_a$. Continuous functions bounded. (8)

Lent Term

1. Differentiation, implies continuity, sums etc. Higher derivatives, polynomials, $\exp$. Chain-rule, derivative of inverse functions, $\log$, x^a , power series. (4)
2. Rolle's theorem, mean value theorem, $\log(1+x)$. Cauchy mean value theorem, L'Hopital, Taylor's theorem, binomial series. (7)
3. $\sin z$, $\cos z$, $\sin(z_1+z_2)$, π , periodicity of $\sin$, $\cos$, $\lim(\sin x/x)$, graphs, $\sin^{-1}$, etc. $\sinh$, $\cosh$, $\sinh^{-1}$, $\cosh^{-1}$. (3)
4. Riemann integral. Upper and lower sums and integrals, conditions for integrability, continuous functions integrable, $|\int f| \leq \int |f|$, subintervals. (3)
5. Indefinite integrals, fundamental theorem of calculus, integration by parts and substitution, Taylor's theorem with integral remainder. (3)
6. Improper integrals of both kinds. Comparison test, Maclaurin's integral test. Euler's limit, integration by parts and substitution. (5)
7. Functions of several variables, limits, continuity, differentiability. (2)

Summer Term

1. Higher partial derivations and optimization. Taylor's theorem. Implicit function theorem, inverse function theorem. (8)

Books

Students are recommended to read books, but not necessarily to buy them in large numbers. One which is worth buying is

J.C. Burkill, A first course in mathematical analysis.

The classic book for this course is

G.H. Hardy, A course in pure mathematics.

For a book with large numbers of examples, try

M. Spiegel, Advanced calculus (Schaum)

Another possibility is

E.G. Phillips, Analysis.

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MT111AnalysisMichaelmas (2 lectures each week)

1. The Real Numbers. Laws of arithmetic, inequalities and proof by induction. The continuity and Archimedean axioms. (3)
2. Limits of Sequences. Definition and examples. The arithmetic of limits. Convergence of monotone bounded sequences. (6)
3. Continuous Functions. Limits of functions and continuity, examples. Sums, products, compositions and inverses. Continuous functions on a closed interval are bounded, attain bounds and take all intermediate values. (7)
4. Differentiation. Definition and examples. Sums, products, compositions and inverses. (4)

Lent (3 lectures each week)

5. Mean Value Theorems. Rolle's theorem. The Mean Value theorem. L'Hopital's rule. Taylor's theorem with remainder. (5)
6. Convergence of Series. Definition and examples. Tests: ratio, comparison. Absolute convergence (rearrangement and multiplication of absolutely convergent series). (8)
7. Power Series. Radius of convergence. Cauchy products. The exponential function. (4)
8. Integration. Definition (Riemann) and examples. Sums and products, integrability of continuous functions. Fundamental theorem of calculus. Integration by parts and substitution. (6)
9. Improper Integrals. Definitions and examples. Absolute convergence. Integral test. (4)

Summer (4 lectures each week)

10. Trigonometric Functions. $\sin x$, $\cos x$ defined by power series. Definition of π . Polar representation of complex number. (4)
11. Convergence of Taylor Series. Integral form of remainder. Logarithmic series, binomial series. (4)
12. Functions of Several Variables. Limits and continuity of functions of two variables. Partial derivatives and differentiability. Chain rule. Optimisation and Lagrange multipliers. (8)

Book List

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|-----------------------|---|
| J.C. Burkill. | A first course in mathematical analysis.
Cambridge University Press, 1962. |
| R. Courant & F. John. | Introduction to calculus and analysis, Volume I.
Wiley, 1965. |
| M.R. Spiegel. | Advanced Calculus. Schaum, 1963. |

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MT112Algebra

As an approximate guide, topics in linear algebra are divided into two kinds: computational (e.g. equations and matrices) and abstract (e.g. vector spaces and linear transformations). Term 1 consists mainly of computational material, with hist of future abstraction, and terms 2 and 3 mainly of abstract material, with reference back to term 1 whenever relevant.

Suitable text books:

1. Thomas A. Whitelaw, 'An Introduction to Linear Algebra', Blackie.
2. A.O. Morris, 'Linear Algebra - an Introduction' VNR New Maths Library.
3. Schaum Outline Series on Linear Algebra.

First term1. Sets (especially of points in $\mathbb{R}^3$).

Sets; subsets, inclusion; $\cup$, $\cap$, ϕ , complement. Functions; 1-1, onto; inverse images, inverse functions. Methods of proof. Cartesian product, $\mathbb{R}^n$, row and column n-tuples.

2. Vector geometry in $\mathbb{R}^n$.

Vector +, scalar multiplication in $\mathbb{R}^3$, $\mathbb{R}^n$. Axioms for real vector space. Linear combinations and dependence. Subspaces; recognition lemma; examples; equations of planes and hyperplanes. Hyperplane lemma. Configurations of planes in $\mathbb{R}^3$, and simultaneous solutions.

3. Linear equations.

Elementary operations on m linear equations in n unknowns. Row echelon form, homogeneous systems. Solution subset/subspace and dimension thereof (using hyperplane lemma). Bases for subspaces of $\mathbb{R}^n$ (e.g. solution spaces) - existence of and dimension.

4. Matrices

Matrix form for equations; products by change of variable, powers, sums, polynomials. Equations as linear transforms $T_A: \mathbb{R}^n \rightarrow \mathbb{R}^m$. Ker, Im, 1-1; nullity, row rank; formulae. $\exists A^{-1} \Leftrightarrow \exists T_A^{-1}$. Elementary matrices, row operations, row echelon form.

5. Transpose and inverse

Transpose, column operations and echelon form. $EAF = I_r$, $A = E^{-1}I_rF^{-1}$; FA and AE . General definition of rank; Inverse computation - rank. Determinant by induction, $\exists A^{-1} \Leftrightarrow \det A \neq 0$, $\det(AB)$, $\det A^{tr}$. Equations and T_A if $\exists A^{-1}$.

6. Diagonalisation

Equations $Ax = b$ and transforming $Ay = c$ into diagonal form, $P^{-1}AP = \Lambda \Rightarrow Ap_i = \lambda_i p_i$ and $\Leftarrow$ if $\exists P^{-1}$. Finding λ 's and p 's by characteristic equation. Differential equations. Distinct eigenvalues $\Rightarrow$ indep eigenvectors.

Second term7. Algebraic structures

Binary operations, associativity and commutativity, examples, e.g. $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$, $M_n(\mathbb{R})$. Definition of a group, deductions from axioms. Informal definition of $\mathbb{Z}_n$; $\mathbb{Z}_n^*$ is a group $\Leftrightarrow$ is prime. definition of a field; $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}$, $\mathbb{Z}_p$. Systems of linear equations over fields, e.g. $\mathbb{Z}_p$. Fundamental theorem of algebra. Equivalence relations and classes, $\mathbb{Z}_n$.

8. Vector spaces over a field k

Definitions and examples, subspaces. Linear dependence, spanning sets, finite dimension. Bases, co-ordinates, k^n . Existence of bases and their properties. Change of co-ordinates. $\cap$ and $+$ of subspaces, direct sum.

9. Linear maps $f: U \rightarrow V$

Definitions and examples, isomorphism. Kernel and image, nullity and rank. Cosets of kernel and solutions of $f(x) = b$. Matrix of a linear map, change of bases, equivalence of matrices. Canonical form, c.f. EAF = I_r .

10. Linear operators $f: V \rightarrow V$

Similarity of matrices, change of bases. Calculation of eigenvalues, characteristic polynomial, geometric multiplicity of eigenvalues.

11. Orthogonal operators

$\mathbb{R}^n$ with inner product, orthonormal bases and co-ordinates. Orthogonal complement, Gram-Schmidt process. Orthogonal operators and matrices 2×2 . 3×3 rotation matrices, orthogonal equivalence.

12. Symmetric operators

Symmetric operators, orthogonal diagonalisation. Spectral theorem.

Third term13. Real quadratic forms

Definitions, representation by symmetric matrices. Arbitrary change of co-ordinates, congruence of matrices, sums of squares. Rank, signature, Sylvester's law of inertia. Orthogonal change of co-ordinates, principal axis theorem. Quadrics. Positive definiteness, simultaneous diagonalisation

August 1984

MT114MethodsMichaelmas Term

1. First order o.d. equations (separable, homogeneous, linear, Bernoulli and useful substitutions). (4)
- 2.* Integration revision (by parts, reduction formulae, partial fractions, reduction to standard forms etc.). Also hyperbolic functions, inverse circular functions. (3)
3. n^{th} order linear equations with constant coefficients. Applications, including the linear harmonic oscillator, resonance, etc. (6)
- 4.* Curve sketching in cartesians and plane polar coordinates. Conics, quadric surfaces, cylindrical and spherical coordinates. (2)
5. Partial differentiation. Chain rule, total derivative, change of variable, grad ϕ , directional derivative, normals to surfaces, higher derivatives, exact equations. (5)

Lent Term

Multiple Integrals: Riemann definition, area integrals, changing the order of integration, volume integrals, general definition, surface integrals, line integrals, applications.

Green's theorem: application to potential theory.

Books

- Fraleigh, John B., Calculus with analytical geometry.
(Addison Wesley)
- Piaggio, Differential Equations, (Bell)
- Lederman, W., Multiple Integrals
Library of Mathematics (Routledge & Kegan Paul)
- Gillespie, R.P., Integration
University Mathematical Texts (Oliver & Boyd)

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MT115 - Probability Theory

This course provides an introduction to the ideas and techniques of elementary probability theory and as such serves as the basis on which the second year courses MT220/221 Mathematical Statistics and MT222 Probability and Random Processes are built. There are two lectures and one examples class per week during the Lent Term.

Probability Spaces

The ideas of chance, uncertainty and probability experiments. Sample spaces and events. The outcomes of an experiment as points in a sample space. The use of set-theoretic notation and ideas to handle descriptions of events.

Probability measures. The addition rule.

Finite sample spaces. The equally-likely probability measure. Urn models and combinatorics.

Conditional probability. Multiplication rule. Theorem of total probability and Bayes' Rule. Independence.

Probability Distributions

Random variables (r.v.'s) defined as mappings from the sample space to the real line. Discrete and continuous r.v.'s. Probability mass functions, density functions and distribution functions. Standard examples of r.v.'s.

Moments of distributions. Interpretation of mean and variance. Chebychev's inequality.

Probability generating functions and their applications.

Repeated Trials and Independent Random Variables

Independent trials and construction of a product measure.

Sequences of Bernoulli trials and Bernoulli's Theorem.

Joint probability distributions and independent r.v.'s. Sums of independent r.v.'s.

The weak law of large numbers.

MT115 - Numerical Analysis and Computing Syllabus.

(Summer term, 2 lectures per week)

Course AFortran programming and numerical methods

This course is given for those students who have not attended a first year Computer Science course involving programming. No credit will be given to students who have attended a Computer Science course involving programming and who answer examination questions on this course.

The course will use FORTRAN 77 which will be implemented on the CYBER interactive computing system. The fundamentals of FORTRAN 77 will be presented up to the level of procedures. Programs will be written to carry out numerical integration, solve nonlinear equations and systems of linear equations. Students will themselves run programs on the CYBER.

Recommended Text Book BALFOUR, A. and MARWICK, D.H. Programming in Standard FORTRAN 77. Heinemann Educational Books.

Course BNumerical Analysis

This course is given for those students who have attended a first year Computer Science course.

The course introduces some basic processes and concepts of Numerical Analysis. Numerical Methods, Dahlquist and Bjork, Ch.1-3, is an excellent reference for this course.

Introduction

The need for numerical methods. Approximation and iteration. Mathematical and numerical problems, existence of solutions and algorithms: definitions and examples. Analysis of an unstable recursion formula.

Sources of error and estimation of error in numerical results

Absolute and relative error. Machine arithmetic. Basic concepts in error analysis, condition and condition numbers.

Numerical use of series

Acceleration of convergence, semi-convergent series.

(7)

MT116Mechanics1. Vector Algebra (7)

Geometric vectors as equivalence classes of directed line segments; sum and scalar multiplication of vectors; dot and cross products; triple scalar and triple vector products; sets of basis vectors and reciprocal sets; differentiation of a vector function of a scalar variable; velocities and accelerations in plane polar co-ordinates.

2. The dynamics of a single particle (6)

Newton's Law of motion and superposition of forces, inertial frames; particle motion in uniform field with linear resistance; soluble problems of one-dimensional motion. The one dimensional harmonic oscillator; forced harmonic motion.

3. Motion in a central field - orbits (7)

The motion lies in a plane; the general harmonic oscillator. Kepler's observation and models of planetary motion, gravitation. The radial equation, the path equation, inverse square law, the two body problem, relative motion. Orbital transfer.

4. The kinematics of many-particle systems (5)

Forces of constraint, generalised co-ordinates; holonomic systems, the rigid body; existence of angular velocity of a rigid body; velocity diagrams.

5. Linear and angular momentum, kinetic energy (4)

The calculation P , H , T for systems in terms of their generalised co-ordinates. (The general case of finding H not covered at this stage); moments of inertia, theorems of parallel and perpendicular axes.

6. The conservation principles of mechanics (8)

Internal and external forces, Newton's third law; linear momentum principle; motion of the centre of mass; the rocket; angular momentum principle for fixed points and G , the modified A.M.P.; equations of rigid body motion; energy principle; examples.

7. Three dimensional rigid body motion (3)

Principal axes, the general calculation of H for a rigid body; the equation for the motion of the axis of a spinning top; steady precession about the vertical; the force of the pivot.

Books:

Chester, W.	Mechanics	George Allen & Unwin
Rutherford D.E.	Vector Methods	Oliver & Boyd.
Smith, R.C. & P.	Mechanics	J. Wiley & Sons.
Smith, G.D.	Vector Analysis including the Dynamics of a Rigid Body.	Oxford.

The first of these books is highly recommended for the course.

August 1984

MT118 - AlgebraTerm 1 (2 lectures a week)

Basic field properties of $\mathbb{R}$ (the real numbers), $\mathbb{Q}$ (rationals) and $\mathbb{C}$ (complex numbers). Construction of $\mathbb{Z}_n$. Elementary theory of polynomials. (4)

Linear equations. Row-echelon form. Solutions of linear equations. (4)

Matrices as shorthand for coefficients of system of linear equations. Sum and product of matrices. (4)

Determinants. Relation to solving linear equations. Basic properties. (4)

Classical solid geometry in $\mathbb{R}^3$ (lines, planes, distance, angle, etc.). Examples in higher dimensions. (2)

Term 2 (2 lectures a week)

Vector spaces over a field ($\mathbb{R}$ or $\mathbb{C}$). Axioms, examples, linear dependence, bases, co-ordinates, dimension. (6)

Subspaces, intersection, sum. (2)

Linear maps. Kernel and image. Matrix of a linear map. Change of basis. Rank. (6)

Eigenvalues and eigenvectors. Characteristic polynomial. Diagonalisation of matrices. (4)

Term 3 (2 lectures a week for Computing & Information Systems students & for Joint Honours Mathematics & Physics Students.)

Real quadratic forms. Matrix notation. Reduction to sum and difference of squares. (2)

Orthogonal reduction of quadratic forms including (ordinary) scalar product on $\mathbb{R}^n$ and orthonormal sets of vectors. (4)

Term 3 (2 lectures a week for Joint Honours Computer Science and Mathematics Students only.)

Logic Introduction to logic. The idea of a proposition and its interpretation in a context. Truth tables.

Arguments, logical consequence and tautologies. Testing for logical consequence, Beth trees.

The idea of a formal proof and the axiomatic method. Examples from Peano arithmetic. (6)

Books:

Birkhoff and MacLane	Survey of Modern Algebra	(Macmillan)
Tropper	Linear Algebra	(Nelson)

Second Year Courses

MT210/211	Analysis
MT212/213	Algebra
MT214/215	Methods
MT216/217	Hydrodynamics and Vector Field Theory
MT218	Waves and Electromagnetism
MT220/221	Mathematical Statistics
MT222	Probability and Random Processes
MT224	Logic
MT226/227	Numerical Analysis

July 1984

MT210/MT211 Analysis

In these courses the ideas developed in first year Analysis are extended to functions of a complex variable. This on the one hand provides a deeper insight into the more elementary real variable analysis of first year, whilst on the other hand introduces more sophisticated ideas, including topology, contour integration and uniform convergence. The courses culminate in a treatment of conformal mapping, a subject which has applications in the physical sciences.

MT210 SyllabusMichaelmas Term

1. Uniform convergence of sequences and series of functions. Review of basic convergence results, general principle of convergence. Uniform convergence and continuity, term-by-term differentiation and integration. General principle of uniform convergence and the Weierstrass M-test. (6)

2. Topology of the complex plane. Open, path-connected, closed and compact sets. Continuous functions, intermediate value theorem, continuous image of a compact set is compact. Exponential, logarithmic, circular and hyperbolic functions. Branches of logarithm and general powers. (7)

3. Differentiation. Complex differentiability and its characterisation using the Cauchy-Riemann equations. $f' = 0$ implies that f is constant on a domain. (3)

4. Integration. Definition of contour integral and calculation from the definition. The fundamental theorem of calculus. (3)

Lent Term

1. The Contour Integral. Basic properties for C^1 integrands and piecewise smooth paths. (2)

2. Cauchy's Theorem. Traditional statement. Proof by differentiating under the integral sign of a fixed end point homotopy. (4)

3. Residue Calculus. Residue theorem. Usual applications to real definite integrals. (4)

4. Analytic Functions. Cauchy's integral formula. Taylor series. Liouville's theorem. Max-mod principle. The argument principle. Rouché's theorem. Morera's theorem. (5)

5. Meromorphic Functions. Laurent expansion. Mittag-Leffler expansion. Application to summation of series. (5)

Summer term

Conformal mappings. Definitions and basic properties. The Riemann sphere, conformal automorphisms of the sphere - the Möbius transformations. Mapping properties of powers and roots and Möbius transformations. Mapping properties of $\exp$, $\log$, $\sin \tan$, $z \pm (z^2 - 1)$.

(Time permitting) Conformal automorphisms of the disc, uniqueness of conformal mappings. (8)

Books

- J.C. Burkill and H. Burkill, A second course in mathematical analysis, Cambridge.
- K. Knopp, Introduction to the theory of functions, Dover. Theory of functions I, Dover, Problem book in the theory of functions I, Dover.
- M.R. Spiegel, Complex variables, Schaum outline, McGraw-Hill.
- I. Stewart and D.O. Tall, Complex Analysis, Cambridge.
- D.O. Tall, Functions of a complex variable, Routledge and Kegan Paul.

MT211 SyllabusMichaelmas Term

Review of differential calculus. Taylor's theorem, estimation of remainder. Power series, convergence. (6)

Functions of several (i.e. 2) real variables. Taylor's theorem. Optimization with constraints. Functions from $\mathbb{R}^2$ to $\mathbb{R}^2$, Jacobian matrix. (7)

Review of integration. Multiple integrals. Line and surface integrals, Green's theorem. (7)

Lent TermComplex Variable.

1. Functions of a complex variable. Differentiation. The Cauchy-Riemann equations. Analytic functions on a domain. The exponential and logarithmic functions and their relation to trigonometric functions. (4)
2. Contours. Integration along a contour. Cauchy's theorem, proved using Green's theorem. (4)
3. Cauchy's integral formula. Taylor's theorem, Liouville's theorem. Maximum modulus principle. (4)
4. Laurent's theorem. Singularities, poles, isolated essential singularities. Residues and Cauchy's residue theorem. (4)
5. Applications to evaluation of real integrals and summation of series. (2)
6. Numbers of zeros and poles. Rouché's theorem, argument principle. Locating the roots of a polynomial. (2)

Note. Detailed methods of calculating residues, and more advanced applications to evaluation of real integrals, are treated in MT214/5.

Summer TermConformal Mappings.

Definitions and basic properties of Möbius transformations. Mapping properties of elementary functions. (8)

Books

P.V. O'Neil*	Advanced Calculus	Routledge & Kegan Paul
D.O. Tall	Functions of a complex variable.	Routledge & Kegan Paul
E.G. Phillips	Functions of a complex variable.	Oliver & Boyd
J.C. Burkill & H. Burkill	A second course in mathematical analysis.	Cambridge
L.V. Ahlfors	Complex Analysis	McGraw-Hill

*O'Neil is strongly recommended for the Michaelmas Term. Analysis books purchased for 110 are alternatives for this term.

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MT212/MT213 Algebra

These courses deal with the theory of groups & rings. The Michaelmas term will run as reading courses with no lectures.

The theory of groups occupies the first term of the MT212 course. Groups arise naturally from the study of the symmetries of regular solids and from the theory of equations. The basic properties of groups are investigated; the classification of finitely generated Abelian groups and Sylow's theorems are discussed with many examples. The second term of the MT212 course is devoted to ring theory. Their defining characteristic is that, in some sense, their elements can be added, subtracted and multiplied together. Examples of rings are sets of matrices or polynomials or functions (the Gaussian integers and exotic examples like the quaternions are also studied). In the third term, some geometrical applications of group theory are given. Rigid motions of the plane are studied and the classification of frieze and wallpaper patterns is obtained.

MT213 has much material in common with MT212 but the more abstract ideas are avoided and greater emphasis is placed on computation and examples. For example in the study of rings during the second term, particular emphasis is placed on polynomial rings and simple examples of those rings which arise in number theory. In the third term the Jordan canonical form is studied.

MT212 SyllabusMichaelmas Term: Group Theory

This term runs as a reading course. There will be no lectures although examples will be assigned and tutorials provided. The course text is R. Allenby, Rings, Fields and Groups, E. Arnold.

Topics include: axiomatics; subgroups; homomorphisms, normal subgroups and quotient groups; groups acting on sets; Sylow's theorems about subgroups of prime power order.

Lent Term: Ring Theory

(a) Axioms for a ring, examples, elementary deductions from the axioms. Many examples including $\mathbb{R}[x]$. Zero divisors, integral domains, cancellation. Units. Division rings, the quaternions, fields, the field of fractions of an integral domain. (3)

(b) Ring homomorphisms, kernel and image. Ideals, subrings. Quotient rings. The isomorphism theorem $R/\ker\theta \cong \text{im}\theta$. (3)

Ideals in commutative rings, prime ideals, maximal ideals, principal ideals. Maximal implies prime, I is prime (resp. maximal) $\leftrightarrow R/I$ is an integral domain (resp. field). (4)

(c) Factorisation in integral domains. Divisibility, prime, irreducible, associates. Unique factorisation domains. Every principal ideal domain is a unique factorisation domain. Examples including $\mathbb{P}[x]$. (4)

(d) Euclidean domains. Highest common factor, Euclidean algorithm. Euclidean valuations. Every Euclidean domain is a principal ideal domain. Examples including $\mathbb{Z}$, $\mathbb{P}[x]$, $\mathbb{Z}[i]$. (3)

(e) Extension fields. Algebraic and transcendental elements. Evaluation homomorphism $f \rightarrow f(\alpha)$. Kronecker's theorem: every non-constant polynomial in $F[x]$ has a root in some extension field of F . Examples including finite fields. (2)

(f) Review. (1)

Summer Term

Isometries of $\mathbb{R}^2$. Classification. (4)
 Classification of strip patterns and wallpaper patterns in $\mathbb{R}^2$. (4)

or

Jordan canonical form. (8)

Books

C.R.J. Clapham. Introduction to Abstract Algebra, R.K.P.
 (Useful as preliminary reading.)

J. Fraleigh. A First Course in Abstract Algebra, Addison-Wesley.
 (Covers most of material: Chaps. 23-25 cover L material.)

J.A. Green. Sets and Groups, R.K.P.
 (A more elementary introduction to the Michaelmas term.)

W. Ledermann, Introduction to Group Theory, Longman.

G. Birkhoff & S. MacLane. A Survey of Modern Algebra, Macmillan.

I.N. Herstein. Topics in Algebra, Wiley.

MT213 Syllabus

Michaelmas Term

Groups.

Lectures this term are replaced by a reading course based on Part I of the book "A First Course in Abstract Algebra" by J. Fraleigh (Addison-Wesley, paperback).

Lent Term

Rings

1. Basic Theory I.

Definition and examples of rings, subrings, including $\mathbb{Z}_n$ and direct sums. Elementary deductions from the ring axioms, zero divisors, connection with cancellation, integral domains, units, fields, every finite integral domain is a field. (6)

2. Euclidean Rings.

Definition of Euclidean ring, examples $\mathbb{Z}$, $\mathbb{Z}[i]$, $\mathbb{Z}[\sqrt{2}]$, $\mathbb{F}[x]$.
 Recognising units in Euc. rings. Ideals in Euc. rings, every ideal in a
 Euc. ring is principal. Highest common factors, existence in Euc. rings;
 calculation of h.c.f. in $\mathbb{Z}$, $\mathbb{Z}[i]$, $\mathbb{Z}[\sqrt{2}]$ and $\mathbb{F}[x]$. Roots of
 polynomials and the Remainder theorem. (5)

3. Factorization in Principal Ideal Domains

Definition of a P.I.D. Associates and irreducibles, examples of
 calculating irreducibles, complete description of the irreducibles of
 $\mathbb{Z}[i]$, calculating irreducibles in $\mathbb{R}[x]$, $\mathbb{C}[x]$, $\mathbb{Q}[x]$, $\mathbb{Z}_p[x]$, etc. Proof
 of the unique Factorization Theorem for P.I.D.s, with examples.
 Application to some diophantine equations of the form $x^3 = y^2 + k^2$. (6)

4. Basic Theory II.

Homomorphisms, kernel, image, ideals, all in an arbitrary ring R .
 Construction of the quotient ring, and the Basic Isomorphism Theorem.
 Explicit computation of some quotient rings. (3)

Summer Term

Jordan canonical form for matrices. Applications to solving differential
 equations. (8)

Books

- G. Birkhoff & S. MacLane, A Survey of Modern Algebra, MacMillan.
- J. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley.
- I. Herstein, Topics in Algebra (2nd edition), Wiley.

July 1984

MT214/MT215 Mathematical Methods

These courses continue to develop mathematical ideas in the spirit of the first year methods course. The two courses cover the same topics, with the lower level course omitting the more obscure ideas and complicated special cases of any particular method. Most of the course is devoted to the study of ordinary and partial differential equations. Some of the equations, such as Laplace's equation, Bessel's equation and the wave equation, arise in other second year and third year courses. Laplace transformations and Fourier series are developed as useful tools to solve the differential equations. Towards the end of the course integration of a function of a complex variable is studied and ties in with the theory developed in 210 and 211. Various methods for evaluating integrals of a function of a real variable by use of complex variable theory are developed. The summer term is devoted to the study of plane autonomous systems of ordinary differential equations.

MT214 Syllabus**Michaelmas Term****Ordinary differential equations.**

Definition and properties of Laplace transform, including convolution. Ordinary linear d.e.s. with constant coefficients - general nature of solution and application of Laplace transformation to initial value problem. Systems of ordinary linear d.e.'s with constant coefficients - general properties and solution of the normal case by Laplace transforms. Problems of electric circuit theory. (8)

Partial differential equations

First-order equations, characteristics. Simultaneous first-order equations and classification of second-order equations, typified by Laplace, diffusion and wave equations. (4)

Separation of variables, especially 1-dimensional wave equation. Laplace's equation in cylindrical and spherical polars. (3)

Fourier series, Expansion theorem (piecewise differentiable function). Applications to Laplace's equation and heat conduction equation. (5)

Lent Term**Ordinary linear differential equations**

Differential equations in the complex plane. Existence of solution near an ordinary point. General properties of linear d.e.'s - Wronskian, variation of parameters, reduction of order when one solution known. (2)

Series solution about an ordinary point (Airy, Legendre equations). Regular singularity and Frobenius method (Bessel's equation). Properties of P_n , J_n . (7)

Applications of complex integration

Residue of a function at an isolated singularity. Residue theorem and examples on calculation of residues.

Integrals round unit circle and round large semicircle. Jordan's lemma and indentation lemma. $\int_0^{\infty} x^{-1} \sin x \, dx$.

Rotation of path of integration. Integrands with branch points.

Inverse Laplace transform for functions of exponential type and step functions. (9)

Summer Term

Plane autonomous systems, including non-linear examples. (8)

NOTE: Although theorems will be proved, the emphasis of the course is on the application of methods to examples.

Books

The following expensive books are suitable for reference purposes.

Jaeger	Introduction to Applied Mathematics	Oxford
Jeffreys & Jeffreys	Methods of Mathematical Physics	CUP
Sneddon	Elements of Partial Differential Equations	McGraw-Hill
Ahlfors	Complex Analysis	McGraw-Hill
Heading	Ordinary Differential Equations	Elek-Science

Of the less expensive books

Piaggio	Differential Equations	Bell
Burkill	Theory of Ordinary Differential Equations	Oliver & Boyd

are useful for the M and first half of L. The last half of L is covered in

Phillips	Functions of a Complex Variable	Oliver & Boyd
Spiegel	Complex Variables with Introduction to Conformal Mapping - Schaum Outline Series	McGraw-Hill
Spiegel	Laplace Transform - Schaum Outline Series	McGraw-Hill

Open University Course M332

Unit 10 - Calculus of Residue

Unit 14 - Laplace Transforms

Useful for the last part of the M term is

Sneddon	Fourier Series	Routledge
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The S term will follow Chapters 7, 8 of

Simmons	Differential Equations with Applications and Historical Notes	McGraw-Hill
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This book is available in paperback at £4 and is also useful for other parts of the course.

Another very good reference for the S term and related material is

Arnold, V.I.	Ordinary Differential Equations.	M.I.T.
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MT215 SyllabusMichaelmas TermOrdinary differential equations

Revision of first year method of solution of ordinary linear d.e.s. with constant coefficients. Definition and properties of Laplace transform (excluding convolution). Solution of initial value problem by Laplace transformation. Systems of ordinary linear d.e.s. with constant coefficients, solution of normal case by Laplace transforms. Problems of electric circuit theory. (8)

Partial differential equations

Recall first year work on partial differentiation (consult 211 lecturer). First-order equations - geometrical interpretation and characteristics. Statement of 3 main types of second-order equations, exemplified by Laplace, diffusion and wave equations. (3)

Separation of variables, especially 1-dimensional wave equation, Laplace's equation in cylindrical and spherical polars. (4)

Fourier series. Formal calculation of coefficients. Application to Laplace's equation and heat conduction equation. (5)

Lent TermOrdinary differential equations

Differential equations in the complex plane. Linear equations, series solution about an ordinary point (Airy, Legendre equations). Wronskian, solution of second order equation by quadrature when one solution known. Regular singularity and Frobenius method, excluding logarithmic case. (Bessel's equation). Properties of P_n , J_n .

Applications of complex integration

Poles and residues. Residue theorem and calculation of residues. Contour integrals round unit circle and large semicircle. Jordan's lemma. Rotation of path of integration. (9)

Summer Term

Plane autonomous systems (linear equations in the plane). (8)

NOTE: The emphasis in this course should be on the applications of the methods to examples.

Books

The following expensive books are suitable for reference purposes.

Jaeger	Introduction to Applied Mathematics	Oxford
Sneddon	Elements of Partial Differential Equations	McGraw-Hill
Heading	Ordinary Differential Equations	Elek Science
Simmons	Differential Equations with Applications & Historical Notes.	Mc-Graw-Hill

(The S term will closely follows Chapters 7, 8).

For the Lent term the following book is highly recommended.

Zill, D.G.	"A first course in Differential Equations with Coefficients."	Wadsworth
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Of the less expensive books

Piaggio	Differential Equations	Bell
Burkill	Theory of Ordinary Differential Equations	Oliver & Boyd

are used for M and first half of L. The last half of L is covered in

Phillips	Functions of a Complex Variable	Oliver & Boyd
Spiegel	Complex Variables with Introduction to Conformal Mapping, Schaum Outline Series	McGraw-Hill
Spiegel	Laplace Transform, Schaum Outline Series	McGraw-Hill

Open University Course M332
Unit 10 - Calculus of Residues
Unit 14 - Laplace Transforms

Useful for the last part of the M term is

Sneddon	Fourier Series	Routledge
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For the summer term

Schwarzenberger	Elementary Differential Equations	Chapman & Hall
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is recommended.

MT216/MT217 Hydrodynamics and Vector Field Theory

These first courses on fluid motion start with some simple problems that can be tackled using first year ideas. For example, momentum considerations enable one to find the force a fireman needs to exert to keep the hose nozzle at rest. The Euler partial differential equations that describe the motion of an ideal fluid are then derived, with the aid of vector field theory; the operators grad, div and curl and the associated integral identities due to Green and Stokes are introduced as required. MT216 contains a discussion of vorticity and the penetrating theorem of Kelvin, which is omitted in MT217. Some simple irrotational flow solutions are obtained, such as flow past a sphere using ideas from the methods course, and flow past two-dimensional bodies using conformal mappings. In the summer term tensors are considered in MT216 whilst MT217 completes the above material.

MT216 Syllabus

Michaelmas

Distinction between real and ideal fluids. Bernoulli's equation derived from pipe flow without resistance. Selection of ideal fluid problems treated by application of conservation of mass, momentum and energy to fluids in bulk; e.g. Torricelli's theorem, reaction on bend in pipe, deflection of a jet, two-dimensional jet impinging on a plane, Borda mouthpiece. Small sudden enlargement of a pipe. (4)

Description of real fluid effects. Newton's law of viscosity. Reynolds number. (1)

Scalar fields, geometrical and Cartesian forms of gradient. Conservative force as a gradient. $-\text{grad } p$ as force per unit volume. Revision of surface and volume integrals, Green's lemma. Physical interpretation, connection with Archimedes principle. (4)

Vector fields. Velocity field, streamlines and particle paths; steady and unsteady flow. (Streamline type equations are treated in MT214/5). Continuity equation for an incompressible fluid; divergence and the divergence theorem (from Green's lemma). Nature of solenoidal field; stream tubes. Green's theorem, div in general orthogonal co-ordinates. (6)

Local, convective and total rates of change of a scalar quantity. Continuity equation for a compressible fluid. The acceleration vector. Euler equations of motion; illustrative solutions. Bernoulli's equation for steady three-dimensional flow. Boundary condition $\frac{DF}{Dt} = 0$ on $F = 0$. (5)

Lent

Linear momentum, angular momentum. Vorticity as twice the angular velocity. Curl and Stokes's theorem. Vortex lines and tubes. Solenoidal character of vorticity field. Line vortex. Kelvin's theorem; corollaries concerning the motion of vortex lines and permanence of irrotational flow. (6)

Irrotational flow. Uniqueness of velocity in simply-connected regions. Doubly-connected regions, the cyclic constant. Irrotational flow of an incompressible fluid; Laplace's equation. Elementary solutions of Laplace's equation in 2D (using separation of variables). Flow past a cylinder and ellipse. Point and line sources, doublets, line vortices. Brief description of method of images, circle theorem. (6)

Two dimensional flow. Complex potential; the stream function. Boundary condition. Conformal mappings. Flow at incidence past a plate with circulation. Blasius's formula. Axisymmetric solutions of Laplace's equation - flow past a sphere. Other mappings as time permits. (6)

Summer

Summation convention, orthogonal transformations. (1)

Definition of a tensor, elementary results, Kronecker delta and alternating tensor. Determinants. (2)

Symmetric and antisymmetric tensors of second order. Finite rotations, small rotations. Vector calculus. (3)

Isotropic tensors. (2)

Books

Fundamentals of Hydro- & Aeromechanics. Prandtl & Tietjens. Dover.
 Advanced Vector Analysis. Weatherburn. Bell
 (The above two books together cover the first two terms' material).
 Fluid Mechanics Batchelor. Cambridge.
 (A comprehensive book useful for third year viscosity course as well)
 Cartesian Tensors. G. Temple. Methuen
 (for the Summer term).

MT217 Syllabus

Michaelmas

The nature of fluid mechanics; the principles of system of particles mechanics that are to be applied. Ideal fluids; Bernoulli's equation from pipe flow. Some simple problems treated by conservation of mass, momentum, energy; Torricelli's theorem, reaction on bend in a pipe, two-dimensional jet impinging on a plane. (6)

Scalar fields, geometrical and Cartesian forms of gradient. Conservative force as a gradient. $-\text{grad } p$ as force per unit volume. Surface and volume integrals, Green's lemma. Physical interpretation, connection with Archimedes principle. (5)

Vector fields. Velocity field, streamlines and particle paths; steady and unsteady flow. (Streamline type equations are treated in MT214, 215.) Continuity equation for an incompressible fluid; divergence and the divergence theorem (from Green's lemma). Nature of a solenoidal field; stream tubes. Green's theorem, div in other orthogonal co-ordinates. Local, convective and total rates of change of a scalar quantity. Continuity equation for a compressible fluid. (9)

Lent

The acceleration vector. Euler equations of motion; illustrative solutions. Bernoulli's equation for steady three-dimensional flow.

Boundary condition $\frac{DF}{Dt} = 0$ on $F = 0$. (6)

Curl and Stokes's theorem. Irrotational flow. (3)

Irrotational flow. Uniqueness of velocity potential in simply-connected regions. Doubly-connected regions, the cyclic constant. Irrotational flow of an incompressible fluid; Laplace's equation. Elementary solution of Laplace's equation. Flow past a cylinder and a sphere. Point and line sources, doublets. The method of images. (9)

Summer

Two dimensional flow. Complex potential; the stream function. Boundary condition. Conformal mapping. Flow at incidence past a plate with circulation. (8)

Books

The vector field theory occurring in M and L is covered in each of

Advanced Vector Analysis	Weatherburn	Bell
Vector Methods	Rutherford	Oliver & Boyd

The hydrodynamics is covered in		
Fluid Dynamics	Huges & Brighton	Schaum Outline Series, McGraw-Hill

(especially chapters 3 and 6)

Hydrodynamics	D.H. Wilson	Edward Arnold
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(concentrates on the two-dimensional parts of the course).

Other books which may be found useful for background reading, or assisting with particular parts of the hydrodynamics are

Fluid Mechanics	Kaufman	McGraw-Hill
Fundamentals of Hydro and Aeromechanics	Prandtl & Tietjens	Dover.

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MT218 Waves and Electromagnetism.

The Michaelmas term course starts with a study of the theory of small oscillations of mechanical systems with several degrees of freedom. It is found that, for certain systems, there is a normal mode of oscillation corresponding to each degree of freedom. The theory is extended to study vibrating strings, which have an infinite number of normal modes. This is a bridging topic of great importance since it provides an introduction to the motion of deformable bodies and involves the study of partial differential equations and the theory of Fourier series (MT214/215).

In the Lent and Summer terms, attention is transferred to electromagnetism. As in the MT216/217 courses on hydrodynamics, one begins with a description of the physical phenomena and then formulates a set of partial differential equations (Maxwell's equations) which make the description more precise. The course uses the vector field theory developed in 216 and 217 and can only be taken by students attending MT216 or 217. A substantial part of the course is devoted to the physical background of the theory, and there is no explicit dependence on A-level physics.

The theory is developed far enough to start a study of electromagnetic waves in the third year.

SyllabusMichaelmas Oscillations and Waves

Small oscillations of a conservative system. Equilibrium conditions for a system with a finite number of degrees of freedom, stability, quadratic forms for potential and kinetic energies, theory of small oscillations, normal modes, matrix notation used in general theory, (quote results for the simultaneous reduction of two quadratic forms, one being positive definite, taught in the algebra course), examples, problems with given initial conditions. (12)

Transverse vibrations of a stretched string. Potential energy and kinetic energy of a vibrating string; equation of motion via Newton's law of motion. (1)

D'Alembert's solution of the wave equation; energy in travelling waves and reflection at a discontinuity. Solution of the initial boundary value problem for a finite string by the method of characteristics. (3)

Fourier series solution of the initial-boundary value problem (combination of normal modes). Non-uniform string. (4)

Lent and Summer Electromagnetism

Electrostatics. Point charges, inverse square law, C.G.S. units, electrostatic field and potential, volume distributions, convergence of integrals at interior points, Gauss's flux theorem. Poisson's equation, properties of conductors, charge on surface, boundary conditions at surface layer, electrical force on conductor, calculation of fields in simple cases from Gauss's theorem and symmetry, spherically symmetric potential as example relating this method to integral for potential. Electrostatic field energy, electric dipoles and double layers. Spherical harmonics of

positive and negative orders, associated boundary value problems (see MT214/215, term 1 Mathematical Methods). (8)

Dielectric and magnetic materials. Volume distribution of dipoles, polarization vector, integral for field not convergent, equivalent charge distribution definitions of $\underline{E}$ and $\underline{D}$ in interior of a dielectric, brief discussion of molecular properties of dielectric and magnetic materials, electrostatic fields, energy of dielectric, calculation of force on a dielectric. Molecular currents, definitions of $\underline{B}$ and $\underline{H}$ in magnetic materials, boundary conditions. (5)

Steady currents. Electric current, current density, equation of conservation, Ohm's law including chemical e.m.f. etc., generation of heat.

Magnetostatics. Magnetic field of steady current, Ampère's law and its generalization to volume current, surface current, boundary conditions, vector potential, Poisson's equation and its integral for the vector potential, thin wires, Biot-Savart formula, force on a current element and on a moving charge. (8)

Electromagnetic induction and Maxwell's equations. Faraday's law, Ohm's law for moving conductor, Maxwell's fourth equation as differential form of Faraday's law, self and mutual inductances. Inadequacy of Ampère's law for non-steady currents, displacement current and modification of Maxwell's third equation, plane electromagnetic waves, Poynting's Theorem, the Poynting vector, potentials, relation of electrostatic and electromagnetic units. (6)

Books

Michaelmas

Coulson, C.A.	Waves	Oliver & Boyd
<u>Lent and Summer</u>		

Coulson, C.A.	Electricity	Oliver & Boyd
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The course was originally based on this book, so it closely follows the lectures. Good value for money.

Two books which concentrate on the physical interpretation of the mathematics are

Feynmann	Lectures in Physics, Vol.2	Addison-Wesley
	Electricity and Magnetism	McGraw-Hill
	(Berkely Physics Course, Vol.2)	

A lot of insight can be gained by reading these. Feynmann is especially easy to read and would make good preparatory reading in the Christmas vacation.

Two other books are

Kip	Fundamentals of electricity and magnetism.	McGraw-Hill
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(Physics biased. Easy to read. Contains an extensive discussion of units) and

Ferraro

Electromagnetic Theory

Athlone Press

(Maths biased. Dull to read, but contains lots of examples for working through).

Finally two books which go well beyond the course (covering the 3rd year syllabus too) but which are useful for reference are

Becker and Sauter

Electromagnetic Fields and
Interactions.

Blackie

Jackson, J.D.

Classical Electrodynamics.

McGraw-Hill

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MT220/221 Mathematical Statistics

These courses follow similar paths, with MT221 giving a rather gentler treatment of the theory while putting greater emphasis on practical work and applications of the theory. An essential part of MT221 is the continuous assessment which consists of questions on the examples sheets to be handed in at roughly fortnightly intervals. The continuous assessment questions count for 30% of the total course mark and the June examination counts for 70% of the course mark.

One aim of both courses is to develop further some of the ideas and techniques of basic probability theory introduced in MT115 with particular emphasis being placed on the handling of probability distributions and other elements needed in the study of statistics. Many of the notions and methods explored at this stage are fundamental to more advanced courses.

The second aim of the courses is to apply the ideas of probability theory to problems of statistical inference - problems in which we are obliged to draw conclusions about underlying features on the basis of limited data gathered in situations where there is inherently an element of uncertainty. The examples considered are drawn from "real life" and aim to illustrate the wide applicability of the theory as well as the art of employing appropriate statistical methods in the interpretation of data.

Both courses have third year continuation courses.

MT220 Syllabus**Michaelmas Term**

Revision of first year lectures: Sample spaces, Events, probability measures, mass and density functions, Conditional probabilities, Independence, Bernoulli trials, Random variables, joint density functions, conditional density functions, marginal density functions. Independent random variables, Bernoulli, Binomial, geometric and Negative Binomial random variables and their relationships. Multinomial distribution. Poisson distribution.

Distribution functions; functions of random variables and their distributions. Sums of random variables and convolutions. Continuous parametric distributions, Expectations and their properties, Conditional expectations, Variance, Covariance, Correlation coefficient. Moment generating functions and their properties. Central Limit Theorem; Normal approximation to Binomial and Poisson.

Lent Term**Populations, Samples and Statistics:**

Data, random samples, histograms, sample distribution function. Statistics, sample mean, sample variance, maximum and minimum of a sample.

Estimation:

Estimators, unbiasedness, consistency. Point estimation, method of moments, maximum likelihood estimation. Interval estimation, pivotal quantities, central confidence intervals. Distributions related to the Normal - χ^2 , t, F. Applications to confidence intervals for the parameters of a Normal distribution.

Hypothesis Testing:

General principles, types of error, test statistics, level of significance. Tests involving the mean and comparison of means, paired comparisons. Tests involving the variance and comparison of variances. The power function. Neyman-Pearson theory, most powerful tests, parametric and nonparametric examples.

Summer TermLinear Regression:

Dependent and concomitant variables. Linear regression, method of least squares, optimal properties of the least squares estimators, point estimation. Normal theory and inference in linear regression.

Books

- B.W. Lindgren. Statistical Theory (Third edition) Macmillan 1978.
 A.M. Mood, F.A. Graybill and D.C. Boes. Introduction to the Theory of Statistics. McGraw-Hill, 1974.

MT221 SyllabusTerm 1

Choosing at random from a population. Random observations and their distributions. Discrete and continuous distributions, probability density functions. Geometric, Poisson, binomial, uniform, exponential, gamma, Normal distributions.

Random variables, their expectations and variances. Bivariate distributions, independence. Moment generating functions. The Central Limit Theorem. The Normal approximation to the Binomial, the Poisson approximation to the binomial and the Normal approximation to the Poisson distribution.

Sample mean and sample variance. The χ^2 , t and F distributions.

Term 2

Considerations in experimental design and sampling methods.

Estimation: point estimates and their desirable properties; unbiased estimates of population mean and variance; confidence intervals for mean and variance of a Normal population; maximum likelihood estimation.

Hypothesis testing: its relationship to confidence intervals; types of error; level of significance; critical region; operating characteristics; construction of tests. Neyman-Pearson Lemma for simple H_0 and H_1 . Tests for means and variances of one or two Normal populations, and for differences in proportions.

Use of sample correlation coefficient to estimate and test population correlation coefficient (Fisher's z-transformation).

Term 3

Linear regression: dependent and concomitant variables; estimation and testing of regression coefficients; prediction.

χ^2 -nonparametric test: goodness of fit; independence of attributes.

Recommended Text

P.G. Hoel: Introduction to Mathematical Statistics, Wiley.

Background Texts

J.E. Freund: Mathematical Statistics, Prentice-Hall, International Edition.

H.J. Larson: Introduction to Probability Theory and Statistical Inference, Wiley International Edition.

B.W. Lindgren & G.W. McElrath: Introduction to Probability and Statistics, MacMillan.

W. Mendenhall, R.L. Scheaffer & D.D. Wackerly: Mathematical Statistics with Application, Wadsworth.

R.E. Walpole: Introduction to Statistics, Collier MacMillan.

Statistical Tables

H.R. Neave: Elementary Statistical Tables, George Allen & Unwin, 1981, £1.50.

These excellent tables are used throughout the course and in the June examination. You should therefore buy a copy.

Copies of all the above books should be available at

Haigh and Hochland University Bookshop
University Precinct
Oxford Road.

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MT222 Probability and Random Processes

In many areas of science, technology, economics etc., processes arise which are best described not by deterministic mathematical models but by models involving an element of "randomness". Examples include the growth of a population over a period of time, the varying content of a water reservoir, the irregular motion of a particle suspended in a liquid, claims made against an insurance policy etc. Probability theory provides the framework for the mathematical treatment of such processes and the course aims to develop the necessary probabilistic background and to introduce the student to the theory of the simplest types of random processes. Although there is no formal prerequisite, knowledge of the probability content of MT115 or its equivalent will be assumed.

Syllabus1st term

(1) Random Walks. Returns to zero and first passage identity. Recurrence and transience. First return time. Simple Random Walk in n -dimensions. Visits to $k \neq 0$. Difference equations and restricted Random Walks. Gambler's ruin problem. (6)

(2) Probability distributions on the real line. Density functions and distribution functions. Mean, variance. Convergence of distributions. The Poisson Approximation theorem. Proof of the De Moivre-Laplace theorem. (7)

(3) Random Variables. Expectations. Sums of independent random variables. Convolutions. Chebychev Inequality and Weak law of large numbers. Borel-Cantelli Lemmas: Strong law of large numbers for Bernoulli trials. (7)

2nd term

(4) Branching processes. The size of the n^{th} generation. Probability of extinction. (4)

(5) Discrete time Markov Chains. States and transitions. The n -step transition probabilities. (2)

Analysis of 2-state Markov Chain, introduction of the ideas of stationary and limit distributions, connection with mean recurrence time. (2)

Classification of states. First passage identity. Irreducibility. Periodicity. (3)

Markov's theorem and examples (with finite state space) (3)

Periodic, reducible and countable chains. Examples (4)

3rd term

(6) Point processes and counting processes. Poisson process. Basic properties. Waiting time paradox. Connection with uniform distributions. Simulation. Non-homogeneous and compound Poisson processes. (8)

BOOK LISTRecommended Texts

The two books which correspond most closely to the material and level of this course are:

1. R. COLEMAN: Stochastic Processes (Allen and Unwin, 1974).
This is in the "Problem Solvers" series and therein lies its strength and its weakness. The theory is stated rather than proved and explanations are few and far between. One attractive feature is the emphasis it places on the use of conditional expectation which, while perhaps a little more sophisticated than the approach used in the course, is worthy of the study of a good student. However, it has good problems and plenty of interesting, "meaty" worked examples. It is also fairly cheap.
2. E. PARZEN: Stochastic Processes (Holden Day, 1964).
Perhaps the oldest established text for this type of course. It is very extensive and includes many worked examples stressing the relevance of the subject to real-life problems. The theoretical treatment is good and it has the honesty to point out where genuine mathematical difficulties are being sidestepped.

Supplementary Reading

1. L. BREIMAN: Probability and Stochastic Processes (Houghton Mifflin, 1969). A good introductory book with interesting examples.
2. W. FELLER: An Introduction to Probability Theory and Its Applications, (Vol. I, Wiley, 1957). The classic text on (discrete) probability theory. Many excellent examples. Treats parts of the theory of stochastic processes. Always repays study.
3. J.G. KEMENY and J.L. SNELL: Finite Markov Chains (van Nostrand, 1960). Corresponds to only a small part of the course and adopts a very different approach. Try it if you like matrix algebra.
4. S.M. ROSS: Introduction to Probability Models (Academic Press, 1972). An elementary introduction to the subject.
5. Y.A. ROZANOV: Introductory Probability Theory (Prentice-Hall, 1969). Gives a good concise introduction to mathematical probability theory.

More Advanced Reading

The natural continuation of this course is MT372. Two texts which have a fair degree of overlap with both are given below. Both are very good but might be rather overwhelming as first texts.

1. D.R. COX and H.D. MILLER: The Theory of Stochastic Processes (Methuen, 1965) (a new edition has recently appeared in paperback).
2. S. KARLIN and H.M. TAYLOR: A First Course in Stochastic Processes, 2nd edition (Academic Press, 1975).
- 2* S. KARLIN and H.M. TAYLOR: A First Course in Stochastic Processes, 2nd edition (Academic Press, 1975).

Note: 2* is actually a rewriting and drastic extension of about half the material in 2.)

MT224 - Mathematical Logic

What do we mean by saying that a mathematical statement is true? What exactly is a mathematical proof? Can mathematics be reduced to entirely mechanical procedures? Are there fundamental concepts in terms of which all of mathematics can be expressed? These are some of the questions which motivate the study of mathematical logic.

The first part of the course will be devoted to the formalisation of a language, the predicate calculus, in which most mathematics can be expressed. A semantics or set of possible interpretations of this language will be defined. A completely mechanical procedure will be given for proving all the purely logical truths which can be expressed in the language.

The second part of the course turns to an examination of axiomatic set theory, first developed by Cantor a hundred years ago, as a foundation for the whole of mathematics. The central notions of cardinal and ordinal number will be investigated, and the role of particular axioms in mathematics will be studied.

SyllabusMichaelmas

The notion of a formal language: the distinction between syntactics and semantics.

The sentential calculus (SC). Truth tables. Adequacy of sets of connectives. Normal form theorems. Compactness theorem for SC.

The predicated calculus (PC). Structures. Isomorphism. Satisfaction and models. An axiomatic system for PC. The completeness and compactness theorems for PC.

Lent and Summer

Elementary operations on sets and functions.

The notion of the size of an infinite set. Cardinal numbers, Schroder-Bernstein Theorem.

Ordered sets. Well-orderings. Ordinals. Transfinite induction. Russell's paradox and the need for axioms. The Zermelo-Frankel axioms of set theory (ZF) and the development of mathematics within ZF.

The Axiom of choice, its equivalents and consequences.

Books:

H.B. Enderton, A mathematical introduction to logic, Academic Press.

A.G. Hamilton, Numbers, Sets and Axioms, Cambridge University Press.

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MT226/MT227 Numerical Analysis

These courses provide a basic treatment of several of the major and most common problems in numerical analysis, namely, obtaining numerical values for the solutions of non-linear equations, systems of linear equations, definite integrals, ordinary differential equations, etc. They are essentially analytic in nature but elementary linear algebra is frequently used and the need to produce finite, practical methods (algorithms) gives a distinctive flavour. They illuminate and give additional insight to ideas of analysis and linear algebra by using and interpreting results in a practical context and they provide a bridge between analysis and applied mathematics. First year Analysis and a knowledge of matrix algebra are prerequisites for the courses. A good understanding of analysis is required for 226 and the course has either an examples class or a tutorial each week. Some knowledge of Fortran programming is assumed in 227 and further lectures on Fortran will be given as required; 227 has a practical period each week and the practical work will be assessed and combined with the examination mark.

MT226 SyllabusMichaelmas

1. Introduction. Simple algorithms and machine arithmetic, rounding error (briefly). (1)
2. Solution of nonlinear equations. Fixed point iteration - convergence. Location of root by bisection, secant, Newton-Raphson, false position and modified false position. Order of convergence. Zeros of polynomials (Newton's method, Bairstow's method, deflation); ill-conditioned roots. Systems of non-linear equations. (6)
3. Interpolation, Lagrange and Hermite forms. Newton's divided-difference formula. Error terms, convergence. Interpolation at equidistant points. (4)
4. Difference operators and applications. Numerical differentiation. Summation. (2)
5. Approximation of functions; function norms (Weierstrass' theorem). Minimax approximation - error alternation property (examples). Continuous and discrete least squares. Fourier series. Properties and use of orthogonal polynomials (Legendre, Chebyshev, etc.). Algorithms for mathematical functions, briefly. (6)

Lent

6. Quadrature. Trapezium and Simpson's rules with error terms, undetermined coefficients. General Newton-Cotes formulae, order of the error. Error estimates. Adaptive and Romberg rules. Gaussian quadrature. Integration on infinite ranges. Singularities. (6)
7. Introduction to ordinary differential equations. Euler's method and convergence. Modified Euler and second-order Runge-Kutta. General explicit methods (Runge-Kutta, Taylor); programming aspects and error checks. Predictor-corrector methods (Adams-Bashforth type). Stability, connection with convergence. Absolute stability. Inherent instability. (9)

8. Boundary-value problems (as time permits). Shooting and matching at one end, problem of inherent instability. Extension to systems of equations.
(3)

Summer

9. Solution of linear equations. Gaussian elimination (triangular factorization), accuracy of results. Matrix norms, bounds for errors, ill-conditioned equations. Iterative refinement. Elimination methods for special matrices.
(7)

Books

The following course text will be used:

Johnson, L.W. and Riess, R.D. Numerical Analysis. (2nd edition)
Addison-Wesley, 1982.

Also useful for background:

Dahlquist, G. and Bjorck, A. Numerical Methods. Prentice-Hall. Good for general reading but quite advanced.

MT227 Syllabus

Michaelmas term

1. Introduction and general principles.
Evaluation of formulae. Types of error. Description of computer arithmetic. Brief analysis of rounding error in floating point arithmetic. Algorithms and flow-charts.
(4)
2. Non-linear equations. Bisection, secant, modified false-position. Fixed point iterations, Newton-Raphson method. Convergence and order of iteration. Evaluation of square roots. Roots of polynomials (incl. Muller's method), deflation, mention ill-conditioning.
(7)
3. Finite differences, definitions and relations between operators. Detection of errors in difference tables. Simple difference equations.
(2)
4. Polynomial interpolation. Lagrange's formula and error. Newton difference formulae, Everett's formula, relation with Lagrange's formula. Numerical differentiation, derivation of formulae by operators and from interpolation polynomials. Practical use. Trapezium rule from linear interpolation.
(4)
5. Curve fitting; the interpolating spline (cubic only, elementary treatment), brief mention of least-squares approximation.
(2)

Lent term

6. Numerical integration. Derivation of methods from interpolation formulae. Error terms, estimates and end corrections. Repeated rules, error as a function of h . Idea of undetermined coefficients. Adaptive and Romberg rules. Infinite integrals and singular integrands.
(8)

7. Ordinary differential equations. Revision of basic theory. Euler's method and trapezium method. Taylor series methods for initial value problems. Runge-Kutta methods. Multi-step methods (derivation of Adams-Bashforth methods from integration formulae in difference form). Predictor-corrector methods. Remarks about practical application and starting values. Systems of equations. Inherent and other types of instability. (9)

Summer term

8. Review of results from linear algebra, need for systematic numerical methods. Gaussian elimination with pivoting, triangular decomposition. Ill-conditioning.

Matrix and vector norms, simple direct error bounds. Iterative correction. Gaussian elimination for band matrices. (Idea and results of backward error analysis.) (7)

(Nearest) Course textbook: Conte, S.D. and de Boor, C. Elementary Numerical Analysis, McGraw-Hill (paperback).

Also recommended:

Dahlquist and Bjorck, Numerical Methods, Prentice-Hall.
 Hornbeck, R. Numerical Methods, Quantum Publishers Inc.
 Proberg, C.E. An Introduction to Numerical Analysis, Addison-Wesley.
 Scheid, P. Numerical Analysis, Schaum.
 Cheney and Kincaid, Numerical Mathematics and Computing, Brooks-Cole.

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List of Third Year CoursesMichaelmas

310	Functional Analysis I
311	Complex Variable
312	Topology I
314	Coding Theory
315	Field Theory
316	Geometry
317	Algebraic Curves
318	Computational Complexity
320	Mathematical Methods I
321	Mathematical Methods I
322	Viscosity
323	Viscosity
324	Electromagnetism & Special Relativity
326	Thermodynamics
330	Statistical Inference
332	Probability Theory
335	Stochastic Processes & Applications
342	Numerical Linear Algebra
343	Numerical Analysis: Special Topics
346	Project

Lent

350	Functional Analysis II
351	Formal Calculus and Combinatorics
352	Topology II
354	Group Theory
357	Algebraic Number Theory
358	Godel's Incompleteness Theorems
359	Machines
360	Mathematical Methods II
361	Mathematical Methods II
362	Waves
363	Waves
364	Elasticity
370	Statistical Methods
371	Statistical Methods
372	Stochastic Processes
382	Numerical Analysis: Advanced Topics
383	Numerical Analysis: Differential Equations
385	Linear Programming
386	Nonlinear Optimisation

Group	MICHAELMAS	LENT
1	W.11.30,Th.9.30,F.2.00,F.3.00X 316} Geometry Ledgard 317} Algebraic Curves C.Williams	358} Godel's Incomplete. Thms. Wilmer's 359} Machines Paris ~~386} Nonlinear Optimisation McLewin~~
2	M.10.30,W.F.9.30,M.4.00X 314} Coding Theory Webb 315} Field Theory Walker	357} Algebraic Number Theory Coghlan 364} Elasticity Wild
3	Tu.9.30,W.10.30,Th.11.30,Th.12.30X 310} Functional Anal. I Reade 311} Complex Variable Little	354} Group Theory Blackburn 362} Waves Hulme 363} Waves Moss
4	M.9.30,Tu.F.10.30,F.4.00X 326} Thermodynamics Marshall 330} Statistical Inf. Robinson	350} Functional Anal. II 351} Formal Calculus & Combinatorics McCradden Ray
5	Tu.2.00,W.12.30,F.11.30,Tu.300X 312} Topology I Hartley 322} Viscosity Watson } (+ Jones X) 323} Viscosity Moss	352} Topology II 372} Stochastic Processes 385} Linear Programming Hall Eccles Doney
6	M.F.12.30,Th.10.30,Th.4.00X 320} Methods I Jones 321} Methods I Ursell	360} Methods II 361} Methods II Ursell King-Hele
7	M.11.30,Tu.12.30,Th.3.00,Tu.4.00X 318} Comput. Complexity Wilkie 332} Probability Theory Papangelou 335} Stochastic Process. Burrell	382} Numerical Analysis 383} Num. Differential Eqns. J. Williams McLewin
8	M.Th.2.00,Tu.11.30,M.3.00X 324} Electromagnetism Gregory 342} Num. Linear Algebra Baker 343} Numerical Analysis Freeman	370} Statistical Methods 371} Statistical Methods Kyprianou Robinson (+ Bowman X)

MT310Functional Analysis I

The course is an introduction to Functional Analysis, which is the application of the abstract method to analysis. Abstract theorems are proved about abstract objects called Banach Spaces. Their implications for the concrete situation are then considered. A simple abstract idea, such as the Contraction Mapping Principle, can often have non-trivial applications, such as a proof of Existence and Uniqueness of Solutions of Differential Equations. The flavour of the course is a mix of algebra and analysis. Prerequisites are some knowledge of convergence (MT110) and of linear algebra (MT112).

Syllabus1. Banach Spaces.

Infinite dimensional vector spaces. Sequence spaces. Function Spaces. Norms. Convergence. Cauchy sequences. Completeness. Open sets, Closed sets, Dense sets. Infinite series. Absolute convergence. Fourier series.

2. Linear Operators.

Limits. Integrals. Differential Operators, Continuity, Boundedness. Norm of an operator. Banach spaces of operators. Dual Spaces. Isometric Isomorphisms. Compact sets. Uniform continuity. Equivalent norms. All norms on a finite dimensional space are equivalent.

3. Category Theory (Baire).

Category 1, category 2. Baire's theorem. Uniform Boundedness Principle. Existence of continuous nowhere differentiable functions. Existence of continuous functions whose Fourier series diverges.

4. Fourier series.

Fejer's theorem. Mean square convergence for continuous functions.

Books

1. Mathematical Analysis II. Burkill & Burkill, C.U.P.
2. Functional Analysis. Goffman & Pedrick, P.H.

July 1984.

MT311Complex Variable

This is a continuation of MT211, so 211 or 210 is a pre-requisite. Two main topics will be covered. First a discussion of the geometric or mapping properties of analytic functions; this will include some further work on conformal mapping. The second topic is about ways of defining analytic functions other than by power series e.g. indefinite integrals, integrals with a parameter. Both topics will be used to define the Jacobian elliptic functions. These are doubly periodic functions which can be interpolated between the trigonometric and hyperbolic functions.

Syllabus

1. Point set topology. Topology of $\mathbb{R}^n$ ($n \leq 4$), compactness and uniform continuity. Proofs will be given for $n = 3$ and geometric arguments allowed where these are not misleading. (4)
2. Line integral theorems. Revision. Integration by parts and by substitution. Continuous and analytic dependence of line integrals on parameters, indefinite integrals. Winding numbers, branches of log and arg. Cauchy's theorem and the residue theorem revised and extended. (9)
3. Convergence. Singular integrals and series of functions. Weierstrass M-test. Principal values of $\sin^{-1} \tan^{-1}$ etc. (4)
4. Mapping theorems. The argument principle and Rouché's theorem revised and extended. Multiplicity and valence. Conformal mappings. Symmetry and Schwarz reflection principle. (9)
5. Inversion of integrals. Definition of $\sin$ by inverting the integral for $\sin^{-1}$. Analogous procedure with elliptic integrals to define the Jacobian elliptic functions. (4)

Books

"Function of a Complex Variable", E.G. Phillips (Oliver & Boyd).

"Elements of Complex Variables", L.L. Pennisi (Holt Reinhart Winston).

"Analytic Function Theory, E. Hille (Ginn).

July 1984.

MT312Topology I

Topology is given only as an upper tier course. It depends to some extent on second-year Analysis material and either level course (210 or 211) is acceptable. Topology is the study of continuity and has grown from attempts to understand ideas in nineteenth century analysis. Today, the basic concepts of topology have permeated virtually the whole of mathematics.

Syllabus

This course has two aspects: (i) the abstract study of continuity, which involves careful and rigorous proofs from axioms, (ii) a geometric flavour, resulting from the fact that one is trying to study properties of geometrical objects that are preserved by continuous deformations of various kinds.

Point Set Topology.

1. Topological Spaces
2. Continuous functions
3. Identification Spaces
4. Compact Spaces
5. Connected Spaces
6. Covering Spaces and the Fundamental Group.

Books

- W.A. Sutherland: Introduction to metric and topological spaces.
Clarendon Press, Oxford, 1975.
- J.R. Munkres: Topology, a first course. Prentice Hall, 1975.

July 1984.

MT314 - Coding Theory

The main subject of the course is the detection and possible correction of errors which arise when messages are transmitted in digital form and the means of communication is imperfect. This forms an important consideration in telecommunications, space technology and computer design. The solution which coding theory provides is to introduce redundant information before transmission so as to protect the original message. The theory draws mainly on algebraic techniques and provides a very good example of an application of pure mathematics in the real world. At a more abstract level the theory connects up with other areas of mathematics, including field theory, combinatorics, geometries and group theory. Codes are widespread: they are used in computer magnetic disc hardware, the Mariner spacecraft, ISBN numbers on books, and in compact audio discs.

Prerequisite: MT212 (it is possible to substitute some directed reading)
Relevant parallel course: MT315.

Syllabus

1. Basic notions Maximum likelihood decoding, Hamming distance, error correction and detection. Perfect codes.
2. Linear codes Weight, minimum distance, generator and parity check matrices, equivalence, dual codes, the syndrome. Hamming codes and their properties. Extending and shortening codes.
3. Weight enumerators Characters, the MacWilliams identities.
4. Hadamard matrices The Mariner spacecraft codes.
5. Cyclic codes Generator and check polynomials, zeros of a code. Hamming codes are cyclic. BCH codes, the Berlekamp decoding algorithm. Reed-Solomon codes.
6. The binary Golay code Construction and properties.
7. Block designs Relationships between the parameters. Steiner systems, the Steiner system $S(5,8,24)$. Incidence matrices and codes.

Books

- J.H. van Lint, Introduction to coding theory, Springer Graduate Texts in Mathematics no.86
- P.J. Cameron and J.H. van Lint, Graphs, codes and designs, LMS lecture notes no.43, Cambridge U.P.
- F.J. MacWilliams and N.J.A. Sloane, The theory of error-correcting codes, Parts I and II. North Holland 1977.

July 1984

MT315Field Theory

This is a classical and elegant branch of algebra, with close links to number theory (MT357). Ideas from MT212/213 about factorisation of polynomials will be used, but ideal theory and quotient rings are not necessary. Some knowledge of the basic ideas of group theory (e.g. symmetric groups, normal subgroups) will also be useful - this will be reviewed as needed.

The main object of the course is to understand and use Galois theory, which links group theory with polynomial equations. Among other applications, this theory explains why there is no general formula for solving a fifth-degree polynomial equation which is analogous to the standard formula for the roots of a quadratic. The course will also visit some of the interesting sideshows of the subject, such as the impossibility of "squaring the circle" and the existence of finite fields of prime power order, some of which are used in coding theory (MT314).

Syllabus

1. Fields. Characteristic of a field, subfields, prime subfield. Constructing fields (i) as field of fractions (ii) adjoining root of an irreducible polynomial. Examples, including some finite fields. (4)
2. Field extensions. Degree $[L:K]$. Simple algebraic and transcendental extensions. Minimum polynomial. Tests for irreducibility of polynomials. Algebraic and transcendental numbers. Algebraic extensions: the field of algebraic numbers. (5)
3. Applications. (1) Straightedge and compasses constructions. Constructible real numbers. Impossibility of doubling the cube, squaring the circle and trisecting the general angle. (3)
4. Galois theory. The Galois group of L over K . The fixed field of a group of automorphisms. Informal discussion of the Galois correspondence, by means of examples. Conjugate elements over a field; their use in constructing automorphisms. (3)
5. Splitting fields. (Subfields of $\mathbb{C}$). Extension of isomorphism $i:K \rightarrow K'$ to isomorphism $L \rightarrow L'$, where L, L' are splitting fields for $f, i(f)$. Normal extensions; finite extensions are normal if and only if they are splitting fields. The fundamental theorem of Galois theory (proof omitted). Examples. Cyclotomic fields; constructibility of regular polygons. (7)
6. Applications. (2) Solution of polynomial equations by radicals. Cubic by Cardano's method. Solvable groups. Examples of quintic polynomials with insolvable Galois groups. Discriminant and splitting field of cubic.

7. (If time permits.) Applications. (3) Finite fields. Multiplicative group is cyclic. The Frobenius automorphism. Classification of finite fields.

Books

- (1) I.N. Stewart, Galois Theory, Chapman and Hall.
- (2) I.T. Adamson, Introduction to Field Theory, Oliver & Boyd.
- (3) J.B. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley.

Each of these three books covers the whole course. Fraleigh is down to earth and the material follows on naturally from groups and rings. This is probably the most suitable book overall, as the more abstract bits can be skipped without loss. Stewart has good historical and motivating material but is more advanced theoretically - don't be taken in by the chatty style! Adamson is also more of an "upper level" book; it is a reliable self-contained classical treatment of the subject, well explained, but with comparatively few straightforward exercises.

July 1984

MT316Geometry

In school geometry courses, Euclidean plane geometry is studied in great detail. Euclid gave certain axioms that were meant to characterise plane geometry, but for 2,000 years after his work appeared, many people suspected that the "parallel axiom" should be deducible from the other axioms. Early in the 19th century, Gauss, Bolyai, Lobachevski and others proved that this suspicion is not mathematically valid. They showed the existence of "hyperbolic" and "elliptic" geometries (which are non-euclidean), and later in the century important models for these geometries were given by Klein and Poincaré. The course will include some study of these e.g. lengths, angles of triangles, area, etc. in the hyperbolic plane. Also, if time permits, there will be some study of hyperbolic 3-space, which has to-day interactions with many other branches of mathematics.

Book List and Syllabus

There is only one recommended book:
 "Notes on Geometry" by E.G. Rees
 published by Springer, 1983 (£7.85).

Notes

1. Copies of this book are available in Haigh and Hochland.
2. This book is the set of notes of a Second Year Course (at Oxford) and will need "strengthening" a little to constitute MT316.
3. MT316 will cover all Part III (Hyperbolic Geometry) of the book, with some additions, and most of Part II (Projective Geometry), again with additions.
4. The additions will be from sources like those listed under the heading "Further Reading" on page 105 of Professor Rees's book, but there is no one other single book to recommend here.
5. Part I of the book, entitled "Euclidean Geometry" will not be in the MT316 syllabus, but it starts with our MT212 Summer Term material (namely isometries of $\mathbb{R}^n$, specialising to $\mathbb{R}^2$) and continues with isometries of $\mathbb{R}^3$, which are only omitted from the MT212 Summer Term course for reasons of time. Hence, you should also find Part I of the book interesting mathematical reading.

July 1984.

MT317Algebraic Curves

An algebraic curve C in the real (affine) plane $\mathbb{R}^2$ is the set of zeros of a single polynomial in two variables over $\mathbb{R}$:

$$C = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 0, f \in \mathbb{R}[X,Y]\}$$

- a very familiar object.

This course studies the properties of algebraic curves, for the most part in the complex projective plane. The detailed projective geometry and classification of conics and cubics lead on to general questions concerning the singular points of curves, counting intersections of two curves (Bezout's Theorem) and producing projective invariants for curves. The emphasis is on geometrical properties and the only prerequisites are first year linear algebra and a little basic group theory.

Syllabus

This course is intended to be a reasonably self-contained and elementary introduction to the geometry of algebraic curves, for the most part in the complex projective plane $\mathbb{P}_2(\mathbb{C})$. An algebraic curve in the real (affine) plane $\mathbb{R}^2$ is simply the set C of zeros of a single polynomial in two variables over $\mathbb{R}$:

$$C = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 0, f \in \mathbb{R}[X,Y]\}$$

- a very familiar object.

In algebraic geometry, geometrical problems are studied by means of algebra (fields, rings, ideals, ...) and algebraic problems by means of geometry (intersection theory, resolution of singularities, geometrical invariants ...). We shall adopt a geometrical point of view throughout this course, revising and developing the algebra required as we go along.

1. Vector spaces and elementary properties of projective spaces (mainly complex). Propositions of incidence. Homogeneous coordinates. Projective transformations. Relation between (two-dimensional) Euclidean and projective geometry. (6)
2. The complex projective line and plane. Cross-ratio. Duality. (3)
3. Elementary projective geometry of conics in $\mathbb{P}_2(\mathbb{C})$; pole and polar. Classification of conics; cononical forms. (6)

P.T.O.

4. Complex projective plane curves. Reducibility. Tangents. Singular points, multiplicity of a curve at a point. (3)
5. Intersection numbers and Bézout's Theorem. (2)
6. Linear systems of plane curves. Apparent genus. Rationality. (3)
7. Plane cubic curves. Theorem of the Nine Associated Points. Addition on a non-singular cubic. Points of inflexion. Hessian. Classification of cubics in $\mathbb{P}_2(\mathbb{C})$; canonical forms. (6)
8. (Optional topic if time allows). Twisted Cubic. Plücker's Equations. Discussion of blowing-up. (1)

Books

- J.G. Semple & G.T. Kneebone "Algebraic Projective Geometry" O.U.P. 1963.
 J.G. Semple & G.T. Kneebone "Algebraic Curves" O.U.P. 1959.
 A. Seidenberg "Elements of the Theory of Algebraic Curves" Addison-Wesley. 1968

July 1984.

MT318Computational Complexity.

In a subject like mathematics which is so concerned with solving problems, it is natural to ask if it is possible to put a measure on the intrinsic hardness of a problem. For example, for a given number c , how hard is it to decide if c is prime? One such measure might be whether the problem could be answered at all on a computer. Given that it can be solved, more refined measures of hardness might be the storage space used or the time required by the computer.

In this course we shall investigate such classification of hardness and in so doing uncover one of the more interesting open problems in modern mathematics, the $P = NP$ problem.

This course cannot be taken in conjunction with MT359.

Syllabus

Chapter 1. Turing Machines. The basic model for computability. Techniques for constructing TMs. Variants of the basic model. Universal TMs: TMs capable of simulating any other TMs. Unsolvable problems, i.e. problems with no computable solution.

Chapter 2. Complexity measures. Measuring complexity of a problem in terms of the space or time needed for a computation. Linear speed-up, tape compression, reduction in the number of tapes. Hierarchy theorems.

Chapter 3. The $P = NP?$ question. Polynomial time and space. Determinism versus non-determinism. NP-completeness and various practical NP-complete problems. Current state of the $P = NP?$ question.

Bibliography. 'Introduction to automata theory, languages and computation', J.E. Hopcroft and J.D. Ullman (Addison-Wesley, 1979).

MT320 - Mathematical Methods I

In this course, which is a continuation of MT214, functions defined by differential equations are studied by complex variable methods. Students who did well in MT215 may also attend the course; the treatment of the method of Frobenius is not as complete in 215 as in 214 and a little preliminary reading on this topic is advised. Solutions are found in the form of power series, and also of contour integrals. By use of the latter form, solutions valid in one part of the complex plane are extended beyond their original region of definition. This process is known as analytic continuation. Extending the region to the neighbourhood of the point at infinity gives a new type of series called asymptotic. Some special functions, such as the factorial, hypergeometric and Bessel functions are introduced and treated as examples of these methods.

Syllabus

Functions of a complex variables. (Mostly revision).

Cauchy's integral theorem. Taylor and Laurent series. Singularities, including branch points. Riemann surfaces for simple cases. Contour integration: examples. Analytic continuation, particularly of $\int_0^\infty e^{-zu}g(u)du$ by rotating the line of integration. Loop integrals. (7)

Factorial function. Define for $\operatorname{re} z > -1$, hence for other z by recurrence formula. The beta function. Product of $z!$ with $(-z)!$ and with $(z-\frac{1}{2})!$ The function $\psi(z) = \frac{d}{dz} \ln z!$; its expansion in partial fractions. (2)

Differential equations in the complex plane. (Second order, regular coefficients). Solution must be of the form $z^d F_1(z)$, or $z^d \ln z F_2(z) + z^d F_3(z)$, where F_1, F_2, F_3 are Laurent series. Conditions for regular singularity. Equations with 0, 1, 2 regular singularities. The hypergeometric equation as the equation whose only singularities are regular ones at 0, 1, ∞ , with a zero exponent at 0 and 1. The hypergeometric function $F(a, b, c; z)$. The Riemann P-function and its relation to hypergeometric functions. The solution of the hypergeometric equation at 0, 1, ∞ . Integral representation of $F(a, b, c; z)$. Value of $F(a, b, c; 1)$. $F(a, b, c; z)$ in terms of functions of $(1-z)$. (6)

Solutions near a regular singularity when the exponents are equal or differ by an integer. Frobenius's method (mostly revision). Applications, including Bessel's equation. (2)

Confluence of 2 regular singularities to form an irregular singularity at ∞ . The confluent hypergeometric equation. (2)

Asymptotic expansions. Examples, definition. Asymptotic expansion of $\int_0^\infty e^{-zu}g(u)du$, valid in an angle. Stirling's series for $\ln z!$ deduced by putting $g(u) = u(e^u - 1)^{-1}$, for which the integral is $\psi'(z)$.

Behaviour of $z!$ for large $|z|$ and different $\arg z$. (6)

Contour integral solution of differential equations. Single and double contours. Asymptotic expansion of single loop integral. Correspondence with asymptotic series solution. (4)

Books

Functions of a Complex variable and Factorial functions

Titchmarsh, Theory of Functions

or Jeffreys and Jeffreys, Methods of Mathematical Physics

or Whittaker and Watson, Modern Analysis

Asymptotic expansions

Jeffreys and Jeffreys, Ch.17, or Erdelyi, or Copson.

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MT321 - Mathematical Methods I

It is assumed that students have attended MT214 or MT215.

In the first half of the term we shall extend the properties of the Laplace Transform. We shall then be in a position to apply these ideas to the solution of partial differential equations where the Laplace Transform method is probably the most powerful tool available.

In the remainder of the course we shall consider second order linear ordinary differential equations. We shall show that it is not always possible to find convergent power series solutions by the method of Frobenius. This leads us to discuss the properties of asymptotic series; series which are not convergent but are nevertheless useful. Finally, we shall discuss asymptotic series which are generated by integrals.

MT361 is a natural continuation course.

Syllabus

- (i) Laplace Transforms
 Revision of MT214/5 material
 Series expansion of function and transform
 δ -function
 Laplace Transform of J_0 from Bessel's equation for J_n by induction
 Convolution integral
 Inverse transform using Bromwich's integral including branch points
 Applications to partial differential equations. (15)
- (ii) Differential Equations.
 Singular points of linear differential equations, including point at infinity
 Expansions at infinity: essential singularities, asymptotic series, asymptotic behaviour of Laplace-type integrals. (15)

Books

- (i) Laplace Transforms.
 This part of the course is covered by many text books but a recommended book from the Schaum Outline Series is Spiegel, M.R. Theory of Laplace Transforms, McGraw-Hill. This covers all the material with a large number of worked examples and exercises although the theory is at times sketchy.
- (ii) Differential Equations.
 No one book covers all the material but parts of the following are useful for reference purposes:

Jeffreys & Jeffreys	Methods of Mathematical Physics.	C.U.P.
Wasow, W.	Asymptotic Expansions for O.D.E.	Wiley
Murray, J.D.	Asymptotic Analysis.	O.U.P.

MT322 - Viscosity

The prerequisite course is MT216. The portion of that course which deals with fluid mechanics provides a useful background, and the material on tensors is used to accomplish the description of stress (and rate-of-strain) in a continuous medium. Tensors are briefly revised at the start of the course, and the Couette and Poiseuille flow solutions which are obtained in MT216 are recovered from the general equations of motion of a viscous fluid, the Navier-Stokes equations. The equations are a system of non-linear partial differential equations and much of the course is devoted to obtaining special solutions of them and to finding ways in which they may be approximated in a systematic way. In this effort a certain ad hoc ingenuity is required, rather than heavy analysis. The most important practical problem at which the course is aimed is that of external aerodynamics; that is, to calculate the high-speed flow of a viscous fluid past a finite body. This objective will remain unattained, but we may hope to explain the principal features of the solution.

Syllabus

Introduction: Real and ideal fluids, description of stress in a continuous medium. Scope and aims of the course. (1)

Revision of tensors

Kinetics. Rate-of-strain tensor, interpretation of symmetric and antisymmetric parts. Continuity equation.

Analysis of stress. Properties of stress tensor.

Equation of motion; Navier-Stokes equations for Newtonian fluid

Explicit form of these equations in Cartesians. (7)

Simple exact solutions of Navier-Stokes equations. Flows with straight streamlines. Explicit form of equations in curvilinear coordinates.

Rotary viscometer etc. (2)

Dissipation by viscosity (1)

Vorticity equation. Rate of change of circulation. Conditions at a solid boundary. Vortex lines in particular flows. Flow field fixed by vorticity distribution; generation of vorticity at boundaries. (3)

More exact solutions. Similarity solutions: jerked plate, diffusion of vortex sheet and line vortex. Stream functions in plane and axisymmetric flows. Plane stagnation flow, plane flow between non-parallel walls (5)

Dimensional analysis and dynamical similarity. Reynolds number, drag coefficient. Other dimensionless parameters with typical values. (2)

Flow at small Reynolds number. Stokes flow past a sphere, cylinder and other creeping flows. (3)

Flow at large Reynolds number. Derivation of boundary-layer equations. Discussion of separation and formation of wakes. Relevance of potential flow solutions. Displacement and momentum thickness as measures of boundary layer thickness. Blasius solution and other similarity solutions.

Approximate solutions by momentum integral method.

(6)

Books:

Hunt, J.N.	Incompressible fluid mechanics.	Longmans
Rutherford, D.E.	Fluid mechanics (last chapter)	Oliver & Boyd
Any of the following would be found useful for reference purposes:		
Batchelor, G.K.	An introduction to fluid dynamics.	C.U.P.
Landau, L.D. & Lifschitz, E.M.	Fluid Mechanics.	Pergamon
Rosenhead, L.	Laminar boundary layers.	O.U.P.
Schlichting, H.	Boundary layer theory.	Pergamon

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MT323 - Viscosity

This is a lower tier course in fluid mechanics which is designed to follow on from MT217 (or MT216). The main objectives of the course are to derive the Navier Stokes' equations for the viscous flow of an incompressible fluid, and to investigate a few simple solutions. The emphasis throughout the course will be on the physical ideas underlying the mathematics. Although MT363 (Waves) does not depend on any of the material covered in this course, MT323 and MT363 form a natural combination.

Syllabus

Distinction between real and ideal fluids. Nature of transport phenomena. Newton's law and plane Couette flow. (5)

Revision of Stokes' theorem and concept of vorticity, $\omega = \text{curl } \underline{v}$. Analysis of motion near a point in Cartesian; symmetric and anti-symmetric contributions to the linear terms. Rate of strain. Principal axes. (5)

Concept of stress - applied to rectangular cartesian fluid element. Reduction to principal axes. Stress-strain relations. Navier-Stokes' equation for incompressible flow. (6)

Steady one-dimensional solutions of Navier-Stokes' equations. (3)

Kelvin's theorem including viscous term. Vorticity theorems for ideal fluids. (3)

Diffusion of vorticity. Jerked and oscillating plate. (3)

Selection from

- i) Further solutions of Navier-Stokes' equations, possible including two-dimensional flows.
- ii) Energy balance.
- iii) Dimensional analysis and nature of boundary layer. (5)

Books

The course doesn't closely follow any one book.

For general reference, Batchelor's "An introduction to Fluid Dynamics" is comprehensive.

Useful material can be found in the following:

Hughes, "Fluid Dynamics", Ch.3, 4, 5 (Schaum)

Kaufman, "Fluid Mechanics".

Espinazi, "Principles of Fluid Mechanics".

Rutherford, "Fluid Dynamics", especially Ch.V.

(This is probably still out of print, but libraries have copies, and you may get one second hand.)

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MT324 - Electromagnetism and Special Relativity

The first part of this course starts where MT218 left off, that is with Maxwell's equations. After reviewing the basic physical ideas discussed in MT218 we examine the wave like solutions of Maxwell's equations in both conducting and non-conducting media (electromagnetic radiation). An introduction to the theory of radiating aerials is given. The second part of the course deals with the theory of Special Relativity. After an elementary introduction based on simple "thought experiments" we see how Maxwell's equations can be naturally and economically written in relativistic form, and investigate some of the simpler consequences. I think it is fair to say that the mathematics in this course is comparatively straightforward and the difficulties encountered tend to arise when tackling new physical concepts. There is no continuation course. MT218 is a prerequisite.

Syllabus(1) Introduction

Charge, current and the conservation of charge. Maxwell's equations in integral forms and differential form. (2)

(2) Plane electromagnetic waves

Electrostatics, magnetostatics, solutions of problems of prescribed charges and currents respectively. Currents in conducting media, absence of volume charge distribution in the homogeneous case.

Plane polarised monochromatic waves in free space. Elliptic polarisation. Evanescent waves. Plane waves in homogeneous dielectric media. Reflection and refraction problems at plane interfaces. The Brewster angle, total internal reflection.

Plane waves in homogeneous conducting media, skin depth. Reflection problem. (8)

(3) Electromagnetic energy and momentum

The energy equality and its interpretation, the Poynting vector. Energy propagation in plane waves. The momentum equality and its interpretation. The Maxwell stress tensor, momentum propagation in plane waves, radiation pressure. (3)

(4) Electromagnetic fields generated by time harmonic current distributions

The scalar and vector potentials, gauge transformations. The Lorentz gauge, Hertz Potential. Solution of problem.

The far field expansion, radiation conditions, radiated energy.

The multipole expansion, the oscillating electric dipole.

Radiation from antennae and arrays of antennae. (7)

(5) Relativistic formulation

Lorentz transformation in 4-vector form. 4-vectors and tensors. Particle dynamics, 4-momentum.

The electromagnetic tensor, the tensor form of Maxwell's equations. The transformation of $\mathbf{E}$, $\mathbf{H}$, invariant combinations.

The energy momentum tensor. (10)

Books

All the material of the course is contained in each of the following books, both of which are recommended. Jackson has the minor advantage of using the same system of units as will be used in the lectures. For the part of the course dealing with special Relativity, parts of

Rindler "Special Relativity"

and Lawsden "Tensor Calculus and Relativity"

provide useful background reading.

J.D. Jackson. Classical Electrodynamics.

Panofsky and Phillips. Classical Electricity and Magnetism.

MT326 - Thermodynamics

Thermodynamics is a study of the relationship between different forms of energy, in particular work and heat. Its applications are (i) the design of engines (which convert heat to work), (ii) the properties of materials (especially specific heats), (iii) the equilibrium arrangement of any system, and the direction of change if the system is not at equilibrium.

The course is in two parts. In part A (Classical) all materials are treated as continua, and the laws are stated as postulates. The treatment in this section is axiomatic following Caratheodory. In part B (Statistical) the atomic theory of matter is assumed, and the laws of thermodynamics and the fundamental results of Part A are derived from reasonable postulates about the probabilities of microscopic events. There is a major subdivision of Part B. The states available to a system can either be considered discrete (quantum statistical mechanics) or continuous classical statistical mechanics. The main objective is to deduce the properties of materials from knowledge of their molecular nature.

No particular MT course is a prerequisite, but students taking MT326 must not take the second year physics course PC215 (thermal and solid state) as there is significant overlap between these courses.

SyllabusClassical Thermodynamics

Introduction and basic definition. Internal energy U , First law (definition of heat), work in different systems. Zeroth law (definition of temperature). The empirical temperature. Thermometers. Perfect gas temperature scale. Specific heats. Enthalpy H . (3)

Perfect gas (defined by Boyle's Law and Joule's Law). Joule's Experiment. Adiabatic curves for a perfect gas and the effect of irreversibility. Introduction to entropy. (3)

The Second Law. Kelvin statement, Clausius statement and Caratheodory's principle. Derivation of Charles law, the perfect gas temperature scale. Avagadro's law, specific heats of perfect gases. The existence of the Absolute temperature scale valid for all substances (found by coupling any system to a perfect gas system: notes on this). (3)

Entropy. Law of entropy increase for isolated systems. Application to heat engines and refrigerators. Maxwell's relations and the thermodynamic potentials. The Joule-Kelvin expansion. The Helmholtz free energy F discussed briefly (3)

Material properties. Energy equations, general formula for $C_p - C_v$. Equilibrium conditions. The Gibbs potential G is a minimum at constant pressure and temperature derived from the law of entropy increase). Applications: phase equilibrium Clausius Clapeyron equation. Chemical equilibrium. Gibbs law for a mixture of perfect gases. Derivation of the law of Mass Action, application to simple reactions e.g. ammonia synthesis.

(5)

Kinetic Theory of Gases

Kinetic energy, pressure, Maxwell distribution, statement of the H-theorem.

(3)

Statistical Thermodynamics

Some basic ideas from Quantum mechanics. Schrodinger wave equation. Energy levels for a monatomic gas. Microstates, Macrostates, degeneracy. Indistinguishability.

(2)

Bose Einstein and Fermi-Dirac formulae for the thermodynamic weight W of a macrostate. The total thermodynamic weight Ω . The classical limit $g_i \gg N_i$. The Elementary method, with some justification ($\ln \Omega \approx \ln W_{\max}$).

(3)

Boltzmann's equation (refer back to classical entropy and H theorem). The equilibrium macrostate in the classical limit. The partition function Z , the Helmholtz-free energy F .

Application to monatomic gas, equation of state. Factorization property of Z . Diatomic gases. Vibrational and rotational energy levels.

(3)

Books

Adkins, Equilibrium Thermodynamics, McGraw-Hill.

Zemansky, Heat and Thermodynamics, McGraw-Hill.

July 1984

MT330 - Statistical Inference

Statistical Inference is the body of ideas and methods which underly the statistical analysis of data. After reviewing a number of the probability distributions used for modelling real-life situations, we go on to examine the problem of Estimation. We shall compare some of the approaches to this problem and derive asymptotic results leading to "large sample" approximations which are useful in practice. We then consider Hypothesis Testing where interest lies in deciding how well data support pre-conceived theories. Again various approaches are compared and the very powerful generalized likelihood ratio test will be discussed in detail. In the next section, on Bayesian Statistics and Decision Theory, we consider how to take into account any information available apart from the data and any costs or profits which may result from our decisions. Finally, in the section on Nonparametric Statistics, we consider what can be done when we are uncertain about the correct probability model to fit.

The usual pre-requisite for the course is MT220 or a good performance in MT221 with some extra reading. The course provides a useful background to MT370 but is not a pre-requisite for it.

A good preparation for the course would be to read "Comparative Statistical Inference" by Vic Barnett (Wiley) which describes clearly and entertainingly many of the ideas without going into mathematical detail.

Syllabus

- | | |
|-----------------------|---|
| Introduction: | Revision; Exponential Family. |
| Estimation 1: | Fisher Information, Cramer-Rao Lower Bound, efficiency; sufficiency, Rao-Blackwell and Lehmann-Scheffé theorems; asymptotic efficiency. |
| Estimation 2: | Maximum Likelihood; properties, asymptotic distribution. |
| Estimation 3: | Other "classical" methods e.g. method of moments, location invariant estimators. |
| Hypothesis Testing 1: | Power function, critical region, size; Neyman-Pearson Lemma assumed; uniformly most powerful (UMP) tests. |
| Hypothesis Testing 2: | Situations when no UMP test exists; significance tests. |
| Hypothesis Testing 3: | Generalized likelihood ratio test; asymptotic distribution of the test statistic; applications to χ^2 tests. |
| Confidence Intervals: | Random intervals, pivotal quantities, large sample results. |

Bayesian Inference: Prior/posterior distributions; conjugate families; Bayesian estimation and hypothesis testing.

Decision Theory: Utilities; risk, admissibility, minimax criterion; Bayes risk, Bayes solution.

Nonparametric Inference: Nonparametric tests; robustness; asymptotic relative efficiency.

Books

B.W. Lindgren: Statistical Theory, 3rd Edition, Collier-MacMillan International Edition.

S.D. Silvey: Statistical Inference, Chapman and Hall.

A.M. Mood, F.A. Graybill, D.C. Boes: Introduction to the Theory of Statistics, McGraw-Hill International Student Edition.

July 1984

MT332 - Probability Theory

The purpose of this theoretical course is two-fold. To develop a rigorous mathematical basis for probability theory with the degree of sophistication necessary for the advanced theory of random phenomena and the advanced mathematical treatment of statistics; and to prove the fundamental classical theorems which formed the starting point for modern probability.

The course is essentially self-contained and none of the second year probability and statistics courses is a prerequisite. However, familiarity with the language and basic ideas of probability theory (as given in MT220, MT221 or MT222) is desirable, as providing an intuitive understanding of and motivation for the theory.

This course is not a prerequisite for MT372, but does provide a useful background to it.

Syllabus

Topics covered include:

Basic measure theory. The Lebesgue measure. Integration. Abstract measures and their extensions. Product measures.

Probability spaces as measure spaces. Random variables, distributions, expectations.

Convergence of random variables. Sequences and series of independent random variables. Kolmogorov's zero-one law. Kolmogorov's strong law of large numbers.

Convergence in distribution. Characteristic functions. The classical central limit theorem and generalisations.

Texts (a selection):

(1) H.L. ROYDEN	Real Analysis	MacMillan
(2) A.N. KOLMOGOROV & S.V. FOMIN	Introductory Real Analysis	Dover
(3) L.E. CLARKE	Random Variables	Longman Mathematical Texts
(4) K.L. CHUNG	A Course in Probability Theory	Harcourt Brace and World
(5) L. BREIMAN	Probability	Addison-Wesley
(6) J. LAMPERTI	Probability	Benjamin

None of these books will be used as a course text. (1) and (2) contain well written chapters on Measure Theory, while (3)-(6) concentrate on Probability Theory. (3) deals with probabilistic concepts at about the level of this course. (4), (5) and (6) are sophisticated introductions to the advanced theory of probability and contain much material that goes well beyond the scope of the present course. (2) and (3) are modestly priced.

Stochastic Processes and Applications

Stochastic or random processes are concerned with dynamic systems (i.e. systems which develop in time) in which the changes are subject to uncertainty or unpredictability or, more precisely, are governed by probabilistic laws.

For examples of such systems we might consider the varying water level in a dam, the number of people awaiting service at a supermarket checkout, the spread of an infectious disease, the total assets of a motor insurance company or the fluctuations in a stock exchange index.

The course covers the basic theory of simple stochastic processes, emphasising how these are used as models in real-life problems. It is intended to be self-contained and to illustrate how the ideas of probability theory can help in dealing with applied problems which are non-deterministic. The course features continuous assessment which is of importance both in extending the theory and in emphasising modelling (including simulation) to develop a "feel" for stochastic systems.

Although the only formal prerequisite is familiarity with the basic ideas of probability theory (MT115(P)), some familiarity with techniques (the probability component of MT220/221) is desirable.

(Note: The course is barred to students who have taken MT222.)

Syllabus1. Introduction and First Examples

What is a stochastic process? Probability models and conditional probability arguments. Simulations.
Two-state processes. Medium and long-term behaviour.
Random walks on the integers. Returns to the origin.
Generating function methods.

2. Markov Chains

The transition probability matrix. Classification of states. First return times.
Equilibrium distributions.

(Note: Markov chains with a finite and with a countably infinite number of states will be covered. Rigorous derivation of results will usually only be given for the finite-state case.)

3. Counting Processes

The Poisson process. Waiting times and interarrival times. Renewal processes. The renewal equation. Use of Laplace transforms. Reliability theory. The survivor function and the hazard rate.

4. Birth and Death Processes

Infinitesimal transition probabilities and the intensity matrix. The Kolmogorov differential equations. Equilibrium distributions. Applications to queuing systems. Queue disciplines. Single and multi-server queues. Virtual waiting time.

5. Time Series

Stationary and wide-sense stationary (WSS) processes. The autocovariance function. White noise. Standard examples of WSS processes. Moving average and autoregressive processes. Correlograms. Discussion of statistical analysis of "real" time series. Removing trends and cycles. Estimation.

MT335 - BOOK LIST

1. U.N. BHAT: Elements of Applied Stochastic Processes (Wiley, 1972). Plenty of novel examples. Of added interest to those who have attended MT220 and MT221 is its emphasis on the application of statistical inference techniques to stochastic models.
2. L. BREIMAN: Probability and Stochastic Processes (Houghton Mifflin, 1969).
A good introductory text with interesting examples.
3. W. FELLER: An Introduction to Probability Theory and its Applications, Vol. I (Wiley, 1957).
The classic text on (discrete) probability theory with many fascinating examples. Elementary in parts but not necessarily easy.
4. E. PARZEN: Stochastic Processes. (Holden Day, 1964).
A good all-round text for both its theoretical treatment and its many worked examples which stress the relevance of the subject to real-life problems.
5. C. CHATFIELD: The Analysis of Time Series (Chapman and Hall, 3rd ed. 1984).
Just about the most accessible introductory text on time series modelling and analysis.

In addition, many of the topics and applications are mentioned in general texts on Operational Research.

MT342 - Numerical Linear Algebra

This is a course on matrix algebra and its application to a number of practical problems which are of great importance in numerical computations elsewhere. It covers the solution of systems of linear equations, over-determined systems, eigenvalue problems and singular value problems with discussions of applications to curve fitting and linear programming. In addition to its practical value, the course contains much material of mathematical elegance and intrinsic interest. It follows on from first year matrix theory and, to a lesser extent, analysis. Although concerned with the numerical aspects of matrix theory, this is not a numerical analysis course with second year prerequisites. Those parts of numerical analysis which are needed will be introduced when appropriate and they will not form the main emphasis of the course. At the same time it provides a suitable follow on course for those students who have studied MT226 or MT227.

Suitable follow on courses are MT382 Advanced Numerical Analysis (requires MT226) and MT386 Nonlinear Optimisation. This course may not be taken with MT343.

Syllabus

1. Matrix Transformations; elementary transformations. LU factors. Matrix Norms and Stability. Orthogonal Transformations. (2)
2. Linear Equations by elementary transformations and orthogonal reductions. Choleski factorisation and methods for symmetric indefinite systems. (3)
3. Overdetermined systems and least squares solutions. Applications to curve fitting. (2)
4. Generalised inverse. Singular Value Decomposition. (2)
5. Symmetric Eigenvalue Problems. Similarity reduction. Givens and Householder. Sturm Sequences. QR algorithm including brief convergence proof. Eigenvectors-inverse iteration, power method. (5)
6. Generalised eigenvalue problem. Reduction to standard form. QZ algorithm (briefly) (if time permits). (1)
7. SVD algorithm (briefly) (if time permits). (1)
8. Methods for large problems. Conjugate gradients for linear systems. Lanczos and simultaneous iteration for eigenvalue problems. (4)
9. Rank-one modifications. Applications to curve-fitting and least squares. (4)
10. Linear programming. Simplex method - matrix approach using rank-one modifications. (4)

Recommended books

Stewart, G.W. Introduction to Matrix Computations. Academic Press.
 Jacobs, D.A.H. (ed). The State of the Art in Numerical Analysis.
 Academic Press.

(Both books include much more than is required for the course. The second is intended for reference only.)

July 1984.

MT343 - Special Topics in Numerical Analysis

This course covers some important problems in practical computation, including the eigenvalue problem, curve fitting and nonlinear optimisation. It has no second year prerequisites, although it is a suitable follow on course to MT226/7. Suitable courses to follow are MT383 and MT385. This course may not be taken with MT342.

Course assessment will be used.

Syllabus

1. The algebraic eigenvalue problem, mention motivation, theoretical background, power method and inverse iteration. Transformation methods, tri-diagonal form and Sturm sequences, error analysis. (9)
 2. Least squares curve fitting, discrete (l_2) and continuous (L_2) case. Fourier series and orthogonal polynomials. (6)
 3. Statement of the minimax (L_∞ and l_∞) approximation problem. Chebyshev polynomials. Statement of the equi-oscillation theory. Chebyshev expansions. Economisation of power series. l_1 and l_∞ approximation posed as LP problems. (5)
 4. Optimisation of non-linear functions. Introduction, including definition of constrained and unconstrained problems and brief consideration of univariate searches. (1)
- Optimisation of non-linear functions without constraints (steepest descent methods and their disadvantages, Newton method and conjugate gradient methods, etc.) (5)
- Penalty function methods for constrained problems. (2)

Text Books

Most of the material can be found in appropriate parts of:-

- Box, M., Davies, D. and Swann, M., "Nonlinear Optimization Techniques", Oliver & Boyd (London).
- Proberg, C.E. "Introduction to Numerical Analysis", Addison-Wesley.
- Johnson, L.W. and Reiss, R.D., "Numerical Analysis", Addison-Wesley.
- Scheid, P., "Numerical Analysis", Schaum.

July, 1984.

MT346 Project

A project supervised by a member of staff may be substituted for one course. It may be possible to extend the project to become a double project and count for two courses. Further details and a list of possible projects are circulated to second year students during the summer term.

July 1984

MT350Functional Analysis II

Hilbert space is an infinite-dimensional version of n -dimensional Euclidean space. Corresponding to $n \times n$ matrices on $\mathbb{R}^n$, we have the linear operators on Hilbert space. This course is an introduction to such operators, mainly by way of examples and problems (with solutions).

Prerequisites MT310 Functional Analysis I will be a help, but is not essential.

A knowledge of complex variable, which dominates the MT210 and MT211 Analysis courses, is not a prerequisite.

Syllabus

1. Hilbert space. Inequalities of Schwarz and Bessel. Parseval identity. Orthonormal bases. Orthonormalization. Fourier series. Closed subspaces. Riesz representation theorem. Best approximation. Examples.
2. Linear operators. Boundedness and continuity. Norm. Adjoint. Projections. Self-adjoint, unitary, normal operators. Examples.
3. Measure theory. Definition of measure. Completeness of $L^2(0, 1)$.
4. Problems in Hilbert space. This section is not determined in advance, but consists of an assortment of problems (and their solutions) chosen from the following: exponential Hilbert matrix; products of symmetries; multiplication operators; Volterra operator; crinkled arcs; Fourier transform; uncertainty principle; Schur Test; Neumann series; Ergodic theorem; numerical range. Various problems on the Hilbert matrix.

References.

1. R. Beals. Advanced Mathematical Analysis. Chapters 1,2,4. Springer 1973.
2. P.R. Halmos. A Hilbert space problem book. 2nd edition. Springer 1982.
3. M. Choi. Tricks or treats with the Hilbert matrix. American Mathematical Monthly, May 1983.

MT351 - Formal Calculus & Combinatorics

A new course whose pre-requisites are merely elementary first year calculus, and a little linear algebra. The aim is to concentrate on manipulative and calculational skills with differential operators and sequences of polynomials, some of which are hopefully familiar from other branches of mathematics, and others of which arise from more lighthearted sources.

Proposed syllabus (approximate, and subject to possible alteration and rearrangement)

1. Polynomials & differentiation. The ring $P[x]$ for $P = \mathbb{Q}, \mathbb{R}, \mathbb{C}$. Linear operators thereon. $D = \frac{d}{dx}$ and its iterates. $\int = D^{-1}$ and its iterates. The translation e^{aD} . The formal power series ring $P[[D]]$. Power series algebra. Invertibility.
2. Difference Operators. Forward, backward and central difference. Iterates and inverses. Formulae for numerical integration and differentiation. Interpolation. Calculus of finite differences.
3. Δ -operators. Translation invariance. Δ -operators. Umbral notation. e^{aD} . Taylor's theorem. Associated sequences of polynomials. Binomial properties.
4. Calculational techniques. Recurrence formulae. $\frac{d}{dD}$ and Pincherle derivative. Transfer formulae. Rodrigues formulae. Bernoulli formulae.
5. Examples. Factorial polynomials. Exponential polynomials. Bessel, Bell, Bernoulli, Gould, Hermite, Laguerre polynomials. Bernoulli and Stirling numbers.
6. Combinatorics. Generating functions - ordinary, exponential and for associated sequences. Moments. Enumerators. Restricted permutations. Board puzzles. Derivatives of composites. Partitions, trees etc. Hammond operators.

Source books include:

- 'The Umbral Calculus', S. Roman, Academic, '84.
- 'An introduction to combinational analysis', J. Riordan, Wiley, '58.
- 'The calculus of finite differences', L. Milne-Thomson, Chelsea, '60.

July 1984

MT352Topology II

The sphere, the torus and the Klein bottle are examples of topological surfaces or two-dimensional manifolds. The definition says essentially that standing at any point of a surface a short-sighted topologist will be unable to distinguish it from the plane. The aim of the course is to illustrate some of the basic ideas of combinatorial and differential topology by studying surfaces.

Syllabus

1. Topological surfaces. Examples of surfaces: the sphere, torus, klein bottle, projective plane. Construcion of surfaces by identifying the edges of polygons.
2. Combinatorial surfaces. The classification of combinatorial surfaces. Euler characteristic and orientability.
3. Smooth surfaces. Embeddings and immersions of surfaces in $\mathbb{R}^n$. The tangent and normal spaces of an immersed surface.
4. Spherical modifications of surfaces, attracting handles.
5. Immersions in $\mathbb{R}^3$. General position. Triple points of immersions of surfaces in $\mathbb{R}^3$. Banchoff's theorem: number of triple points has same parity as Euler characteristics.

Note Some of the more technical results from differential topology will be quoted without proof.

Prerequisite MT312.

Books

- D.W. Blackett, Elementary topology, Academic Press, 1982.
 D.B. Gauld, Differential topology, an introduction, Marcel Dekker, 1982.
 W.S. Massey, Algebraic topology: an introduction, Harcourt, Brace and World, 1967.
 A.H. Wallace, Differential topology, first steps, Benjamin, 1968.

July 1984.

MT354Group Theory

This course will be in a similar spirit to MT212 and will develop further the ideas of group theory met there. Finite groups will be emphasized.

Syllabus

The basic homomorphism theorems will be treated in full as far as the Jordan-Hölder theorem. There will be a good deal of "Sylow theory", concerned with the more immediate consequences of Sylow's theorem, this being essential for the study of finite groups. However, the Jordan-Hölder theorem draws attention to simple groups. The determination of these is far beyond an introductory course such as this but some indication of how to begin will be given.

July 1984

MT357Algebraic Number Theory

This course is definitely "algebraic". There are two main topics in this course viz. (i) the solving of "simple" diophantine equations (i.e. integer solutions of equations) using the idea of unique factorisation in Euclidean rings, and (ii) the study of congruences i.e. arithmetic (mod m) where m is a positive integer. Two such typical problems are

(i) find all integers solutions of $x^3 = y^2 + 2$

and

(ii) is $x^2 \equiv 79 \pmod{97}$ soluble?

Anybody who has been to MT112 (first year algebra) and either of MT212 or MT213 should have all the background necessary, viz. the definitions of (abelian) group, cyclic group, ring, ideal and also Euclidean ring (although this topic will be revised in some detail).

Some topics may have something in common with those of MT315.

Syllabus

1. Unique factorisation theorem for the integers, primes, simple diophantine equations. (5)
2. Congruences, Z_m , G_m (multiplicative groups of Z_m), Euler totient function $\phi(m)$, theorems of Euler, Fermat, Wilson. (5)
3. Principal ideal domains, unique factorisation theorem, Euclidean rings, applications to diophantine equations. (6)
4. Residues, group structure of G_m , quadratic residues, quadratic reciprocity, more diophantine equations. (8)
5. Pell's equation, ($x^2 - Dy^2 = 1$), or Lagrange's theorem (every positive integer is a sum of 4 squares), as time allows.

Books

There is no "book of the course". The following books may be useful as reference books.

Stewart and Tall. Algebraic Number Theory.

Burton. Elementary Number Theory.

Hardy and Wright. The Theory of Numbers.

Le Veque. Topics in Number Theory.

July 1984

MT358Gödel's Incompleteness Theorems.

Can a computer be programmed to generate all the true statements of mathematics and no false ones? An extraordinary mathematical result, proved by Kurt Gödel in 1931, shows that even if we restrict ourselves to the most fundamental part of mathematics, namely statements about the natural numbers, then the answer to the above question is no. Moreover Gödel's method is such that given any "mechanical procedure" for generating true theorems about the natural numbers we are actually able to construct a true statement of number theory which will not be generated by that procedure.

Gödel's work stands as one of the landmarks of 20th century thought and has a significance which reaches well beyond mathematics. The course will be centred on proofs of Gödel's two incompleteness theorems and will examine both their principal applications in mathematical logic and their philosophical significance.

Prerequisites: MT224 (first term) or a familiarity with the predicate calculus up to the completeness theorem. The course fits well with MT318 although the latter is not a prerequisite.

Syllabus

1. The completeness theorem for the predicate calculus: a short survey.
2. Recursive functions and relations. Basic properties. Closure of recursive relations under bounded quantification. Primitive recursion. Gödel's β -function. Coding of finite sequences. Church's thesis. Recursively enumerable sets and the arithmetic hierarchy.
3. First-order Peano arithmetic and weaker systems. Representability. The representability of recursive functions and relations in first-order arithmetic.
4. Gödel numbering and the arithmetization of logic. The recursiveness of the proof predicate. Gödel's first incompleteness theorem. Tarski's undefinability theorem.
5. Applications of the first incompleteness theorem to show the undecidability of the predicate calculus and other axiom systems. Examples of decidable theories.
6. Gödel's second incompleteness theorem.
7. A brief survey of the main currents in the philosophy of mathematics and of the impact of Gödel's work. The limitations of the axiomatic method.

Books

- | | |
|-----------------|--|
| H.B. Enderton | A mathematical introduction to logic. (especially Ch.3). |
| J.R. Shoenfield | Mathematical Logic (Chs. 4 and 6). |
| G.T. Kneebone | Mathematical Logic and the Foundations of Mathematics. |
| Hao Wang | From Mathematics to Philosophy. |

July 1984.

MT359 - Machines

Although there are no formal mathematical pre-requisites for this course, the subject as a whole requires a reasonable level of mathematical sophistication. For mathematicians the unfamiliarity of the subject matter and the difficulty of some of the problems should compensate for the background depth. It is hoped that computer scientists would gain a new perspective on familiar ideas from the extra abstraction and mathematical precision. The course may also interest those studying linguistics, philosophy and psychology.

The course contains a few key definitions, theorems and constructions. But the heart of the course is a large number of concrete examples.

The course deals with two approaches to the mathematical representation of automata or computing devices. The first notion of finite state machines is strictly finite. The second, Turing machines, although always finite, allows for an indefinite extension of their memory. Turing machines were named after their inventor (who worked here for a few years) and were studied at a time in the 1930's before the development of modern computers. They give a precise mathematical representation of those problems solvable by modern computers and (according to Church's thesis also put forward by Turing) by any conceivable computing device that can be finitely described. Using this precise representation it is possible to show that certain mathematical problems are 'unsolvable'.

Furthermore this gives ways of measuring the intrinsic hardness of solvable problems in terms of the physical properties of the computers needed to solve them. Over the past fifteen years much research work has been carried out on this classification and has revealed a surprising hidden structure. Yet despite so much effort many open questions remain the most famous being the $P = NP$ or Travelling Salesman problem.

There is no second year prerequisite.

This course cannot be taken in conjunction with MT318 or MT358

SyllabusFinite state machines (FSM's)

The basic definition with many examples. Impossibility of multiplying using an FSM. Finite acceptors and the loop theorem. Some non-acceptable languages. Regular languages and Kleene's theorem. Non-deterministic finite acceptors and the Rabin-Scott theorem. The synthesis of FSM's as networks of simple components. (10)

Turing machines

The informal notion of an algorithm or effective procedure. Turing's analysis of this notion and his formulation of Church's thesis. The definition of a Turing machine with examples. A universal Turing machine. The unsolvability of the halting problem for Turing machines.

Classification of the hardness of solvable problems. Non-deterministic Turing machines, the $P = NP$ problem. (17)

Books

There is no single book which is adequate as a text book for this course. Minsky's book can be highly recommended to give the general flavour of the subject of machines and how it relates to other disciplines, it is usually enjoyable to read. Unfortunately it does not have enough routine examples. The book by Hopkin and Moss is perhaps closest in level and scope to the course. It has many examples. The remaining two books are at a more advanced mathematical level and have the drawback of having few routine examples. Nevertheless they may be useful in showing how the subject has developed as a mathematics discipline. It should be kept in mind when reading any of these books that there are usually slight variations in the key definitions used by different authors.

- 1.* M. Minsky. Computation: Finite and Infinite Machines.
Prentice-Hall International (Open University
Text) (1972).
2. A. Salomaa Theory of Automata
Pergamon Press, International Series of Monographs
In Pure and Applied Mathematics, Volume 100 (1969).
3. J.E. Hopcroft Formal Languages and their relation to automata
& J.D. Ullman Addison-Wesley (1969).
- 4.* D. Hopkin & Automata
B. Moss Macmillan Computer Science Series (1976).

* Available in paperback.

MT360 - Mathematical Methods II

A continuation of MT320. Applications of complex analysis to the treatment of differential equations, with particular emphasis on asymptotic estimates. The last section is essentially real-variable theory, although complex-valued functions of the real variable occur, and touches on some general ideas about Hilbert space and reactions between differential and integral equations.

SyllabusBessel's equation

Formal asymptotic series and corresponding contour integral solutions.

Identification in terms of J_ν and Y_ν .

Definition of $H_\nu^{(1)}, (2)$. Asymptotic series for J_ν, Y_ν . Double loop integral giving Poisson's integral for J_ν . Properties of I_ν and K_ν .

(5)

Laplace transforms

Bromwich integral. Transform as an analytic function of the transform variable. Convolution. Methods of evaluation of Bromwich integral, partial fractions, $p \rightarrow \infty$ for $t \rightarrow 0+$, expansions near branch points for $t \rightarrow \infty$.

Applications. Partial differential equations, 1st order, wave equation, heat conduction equation.

(5)

Method of steepest descent

Contour integrals with a large parameter. Saddle points. Application to $n!$, $J_n(n\xi)(\xi' < 1)$.

(5)

Differential equations with a large parameter

Green-Liouville (or WKB) method. Application to $J_n(n\xi)(\xi' < 1)$.

Transition points and Airy functions. $J_n(n\xi)$ near $\xi = 1$.

(4)

Self-adjoint boundary value problems associated with ordinary differential equations

Eigenvalues, eigenfunctions, self-adjointness conditions. Spectrum real, no finite limit point, eigenfunction orthogonal. Construction and properties of Green's functions. The resolvent operator. Properties of integral operators (no proofs).

Expansion and completeness theorems in regular case. Complete orthonormal systems. Parseval identity.

(8)

Book:

Jeffreys, H. and Jeffreys, B.S. Methods of Mathematical Physics.
(C.U.P.)

MT361 - Mathematical Methods II

The pre-requisite for this course is either MT215 or MT214. MT321 is not a pre-requisite. The syllabus is divided into three parts:

- 1). Autonomous systems - a revision and extension of second year material
- 2). Calculus of Variations - here we will be concerned with how to choose a curve $y = y(x)$ so that $\int_a^b f(y', y, x)dx$ is minimized.
- 3). Optimal Control - we will develop the theory in a simple manner, drawing on our knowledge of sections (1) and (2).

Syllabus

- (i) Autonomous Systems of the Second Order
Critical points, stability, linear and non-linear systems, sketching of complete trajectories. (6)
- (ii) Calculus of Variations
Euler-Lagrange equation, extremals with corners, the Legendre condition. Problems with constraints, problems with variable end points. Application of calculus of variations in Applied Mathematics (if time). (9)
- (iii) Control Theory
Statement of control problem for o.d.e.
Discussion of controllability and stability.
Pontryagin maximum principle (proved in a simple manner).
Solution of linear problems with piece-wise continuous controls for time-, cost-, and fuel-minimization.
Transversality conditions for control to target sets. Singular problems (if time). (12)

Books

- (i) Autonomous Systems.
A relatively simple book which covers all the material is Sanchez, D.A. O.D.E. and Stability Theory. Freeman.
- (ii) Calculus of Variations.
All the theory is covered by Clegg Calculus of Variations. Oliver & Boyd
- (iii) Control Theory.
No one book covers all the material but parts of the following books are useful for reference purposes:-

Pontryagin	Mathematical Theory of Optimal Control	Academic Press
Denn, M.N.	Optimization by Variational Methods	McGraw-Hill
Lawden, D.P.	Analytical Methods of Optimization	Scottish Academic Press
Athans & Falb	Optimal Control	McGraw-Hill

MT362 - Waves

Two types of wave dispersion in fluids are considered in this course. Waves on a water surface illustrate frequency dispersion, in which waves of different frequencies - or wavelengths - travel at different speeds. The concept of group velocity leads to some understanding of water wave phenomena such as the appearance of short waves upstream, and longer waves downstream, of an obstacle placed in a stream. Amplitude dispersion - in which waves of different amplitudes travel at different speeds - occurs in a compressible medium such as air. This dispersion can lead to the formation of waves containing a sudden change in flow variables, known as a shock wave in air and a bore on an estuary.

MT216, or a reasonable performance in 217 is a pre-requisite; the tensor material in 216, is not required. MT322 and 323 are related courses but in no sense prerequisites.

Syllabus

Dispersion of Waves. Group velocity. Equations of motion and the linearized boundary condition for surface water waves. Capillary waves and long waves.

Fourier transforms and generalized functions with water wave application in 1 and 2 dimensions. Ship waves. Estimation of wave profile using stationary phase. The radiation condition. Moving oscillating disturbance.

(17)

Compressible Flow

Basic thermodynamics. Thermodynamic equilibrium. Energy equation. Simple flows; Crocco's theorem. Propagation of small disturbances, simple waves, characteristics. One-dimensional piston problems and two-dimensional steady flows. Steepening of wave fronts, formation of shock waves. Conservation laws for shocks. Shock tube.

(13)

Book Waves in Fluids. Lighthill. Cambridge.

Not a course text, but very useful to read in parallel with the course.

July 1984

MT363 - Waves

The course is concerned with waves which occur in fluids. The first half of the term is concerned with waves in estuaries, canals and in the open sea. The remainder of the term is concerned with the flow of gases where the effect of compressibility is important. We shall, for example, consider the propagation of sound waves.

The course follows on from 217 (or 216) and uses some ideas from 215/214 - Fourier Series, separable solutions of partial differential equations. MT323 is not a pre-requisite, but the two courses go naturally together.

SyllabusTidal Waves

Derivation of wave equation for small amplitude long waves in one dimension. D'Alemberts solution. Standing waves, normal modes, solution with given initial conditions, use of fourier series. Reflection and transmission of waves. Energy. Solutions for non-uniform channels. Forced oscillations. Non-linear effects. (9)

Water Waves

Derivation of Laplace's equation in 2-D. Boundary conditions at a free surface. Waves in deep water. Dispersion relationship. Surface tension effects. Transmission of energy. Group velocity. Waves on the interface between two media. (8)

Sound Waves

One dimensional motion. Derivation of wave equation. Boundary conditions. Selected problems in 1-D. General equations for small amplitude motion, 3-D waves, reflection and transmission of sound waves at a rigid plane surface and at the interface between two media. Spherical waves. (6)

Finite amplitude waves

Introduction to compressible gas dynamics, basic thermodynamics. Steady quasi one dimensional flow. Normal shock waves. Unsteady 1-D flows, (if time permits). (7)

BooksRecommended book

WAVES: C.A. COULSON and A. JEFFREY, LONGMANS.

This book covers much of the material in the course.

For reference and general reading

HYDRODYNAMICS: H. LAMB, C.U.P.

PHYSICS OF FLUIDS AND SOLIDS: J.S. TREFIL, PERMAGON.

WATER WAVES: J.S. STOKER, INTERSCIENCE.

MT364 - Elasticity

Equilibrium and oscillations of elastic bodies, restricted almost entirely to small deformation theory. Uses general notions of continuum mechanics, Cartesian tensors, differential equations, a little complex variable theory and some thermodynamics (but MT326 is not a pre-requisite). Applications to beam theory and thin plates and other "two dimensional" systems.

Syllabus

1. General introduction to solid mechanics; relevance to problems in engineering and geophysics.

(1)

2. Extension of a bar; Young's modulus and Poisson's ratio. Euler-Bernoulli theory of flexure; theory of thin beams.

(3)

3. Mechanics of continuous media; analysis of stress and strain; constitutive equations. Linear infinitesimal displacement theory; equilibrium conditions in terms of displacements; strain compatibility relations; Beltrami-Michell equations.

(7)

4. St. Venant theory of beams.

5. Plane strain and generalised plane stress; Airy's stress function; complex stress functions. Examples.

(6)

6. Reciprocal, variational and uniqueness theorems. Statement of thermodynamic results on thermo-elastic relations and isentropic elastic constants, with discussion if time permits. Thermo-elasticity.

(4)

7. Elastic waves; equi-voluminal and irrotational waves. Rayleigh waves.

(3)

Book:

LOVE, A.E.H. The Mathematical Theory of Elasticity. C.U.P.

MT370 - Statistical Methods

In many areas of science, technology, social science and medicine one often wishes to explore the relationships between an observable random response and a number of 'factors' which may influence simultaneously the response. If the influence of all the 'factors' are quantitative the statistical tools developed to explore these relationships are referred to as Regression Analysis, whereas if they are qualitative the corresponding statistical tools are referred to as Analysis of Variance. If some of the factors are qualitative and others quantitative the statistical techniques are called Analysis of Covariance. These three valuable types of statistical techniques can, however, be studied together as special cases of a unified theory of Linear Models. The course starts with a study of estimation and testing in the general Linear Model. Once the general principles and techniques are established, practical applications in the areas of Regression Analysis, Analysis of Variance and Analysis of Covariance are examined in greater detail. (The course includes a number of practical classes in the analysis of data.) The implications of these types of statistical analyses are also used to develop principles of how and why scientific investigators should design experiments. A normal pre-requisite for this course is MT220 or a good performance in MT221, but MT330 is not a pre-requisite.

Syllabus1. Introduction

Multivariate Normal Distribution. Properties of Covariance Matrix. Non-central chi-squared, t , and F -distribution.

2. General Linear Model (design matrix not necessarily of full rank)

(a) Least squares estimation: Least squares estimators (l.s.e.) and their properties, e.g. existence, uniqueness, unbiasedness, variance. Estimable parametric functions. The Gauss-Markov Theorem. Residual sum of squares. Canonical representation.

(b) Normal Theory: Distribution of l.s.e. and residual sum of squares.

(c) Hypothesis testing: Testing values of parametric functions. Derivation of test statistic. Distribution of test statistic under the null and alternative hypotheses. Expected mean squares. Power of the test. Computational considerations.

3. Linear Regression. Simple regression. Testing whether two or more lines are parallel. Multiple regression. Significance tests using ANOVA tables. Polynomial regression. Orthogonal polynomials. Testing for the degree of the polynomial. Examining residuals.4. Analysis of Variance. One way analysis of variance with equal and unequal number of observations. Two way analysis of variance with one observation per cell. Two way analysis of variance with more than one observation per cell.

5. Design of Experiments: Randomisation, precision and replication. Completely randomised designs. Randomised complete block designs. Blocking against two factors with interaction present. Latin square designs. Splitting the treatments sum of squares; use of orthogonal comparisons. Factorial experiments.
6. Analysis of Covariance.

Books

H. Scheffe: Analysis of Variance. J. Wiley.
 B. Wetherill: Intermediate Statistical Methods. Chapman & Hall.
 N.R. Draper & H. Smith: Applied regression Analyses. J. Wiley.
 D.C. Montgomery: Design and Analysis of Experiments. J. Wiley.
 Cochran & Cox: Experimental Designs. J. Wiley.
 Wonnacott & Wonnacott: Regression. A 2nd course in Statistics.
 J. Wiley.

MT371 - Statistical Methods

The course provides an introduction to three important topics in applied statistics: regression, the analysis of variance and sample surveys. The methods introduced are found valuable in many areas of science, technology, social science, management science and medicine. The real-life problems discussed in class will reflect this wide range of applications. To help analyse data, use will be made of an interactive statistical computing package, but no previous experience of computing is necessary.

Regression is concerned with investigating the relationship between a "response" and many "explanatory" variables. Questions of interest to an experimenter can often be usefully formulated in this statistical framework. The analysis of variance is a technique for analysing measurements whose values may be influenced by many "factors" operating simultaneously. Information can be gained on which factors are important and the size of their influence estimated. The theory also has implications about how experiments should be planned and observations taken. The final section of the course on sample surveys introduces some of the mathematical techniques used in opinion polls, market research investigations and government surveys.

The prerequisite of the course is familiarity with the Normal and related distributions (χ^2 , t , F) and with the idea of hypothesis testing (as covered in, for example, MT221 or MT220).

SyllabusIntroduction

Looking at data. Fitting a straight line. Introduction to Minitab (statistical computing package).

General linear model (full rank)

General principles of estimation (least squares, maximum likelihood). Random vectors and the multivariate Normal distribution. General principles of interval estimation and hypothesis testing. The model in deviation form.

Topics in regression

Checking assumptions (residuals, probability plots, transformations, lack-of-fit test). Multiple regression, correlation and choice of explanatory variables. Polynomial regression. Growth curves. Comparison of regression lines. Calibration.

Experimental design models

Linear models of less than full rank, estimable functions, finding the Normal equations. Interval estimation, analysis of variance and hypothesis testing. Completely randomised designs. Randomisation and replication. Multiple comparisons. Randomised block and factorial designs. Interaction. Latin squares. 2^n factorial designs.

Sample Surveys

Simple random sampling from a finite population. Estimating population mean response or proportion possessing an attribute. Stratified sampling; proportional and optimal allocation. Quota sampling. Cluster sampling. Systematic sampling.

Books

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|------------------------------|---|
| Draper, N.R. and Smith, H. | "Applied Regression Analysis".
1966, Wiley. |
| Weisburg, S. | "Applied Linear Regression".
1980, Wiley. |
| Cochran, W.G. and Cox, G.M. | "Experimental Designs".
1957, Wiley. |
| Montgomery, D.C. | "Design and Analysis of Experiments".
Wiley. |
| Steel, R.G.D. & Torrie, J.H. | "Principles and Procedures of Statistics
- a Biometrical Approach".
1980, McGraw-Hill, Kogakusha. |
| Barnett, V. | "Elements of sampling theory".
1974, English University Press. |

MT372 - Stochastic Processes

This course is designed to be a logical follow-up to MT222, but is almost completely self-contained and is equally accessible to those who have taken one or more of MT220, MT221, MT332. The emphasis is on the mathematical analysis of particular models for random (or stochastic) processes. Ideally, the student should have a 'feel' for probability, a sound knowledge of mathematical methods (e.g. differential equations, Laplace transforms) and a grasp of real analysis. An interesting complementary course is MT332.

The course is concerned with stochastic (i.e. random) processes occurring in continuous time. It deals not with the measure-theoretic considerations needed to provide a rigorous mathematical treatment of general stochastic processes, but rather with setting up and analysing certain useful processes. Both probabilistic and analytic techniques are developed.

Syllabus

Continuous time Markov chains. Illustration of the fundamental analytic techniques by means of the Poisson process. The differential matrix of a Markov chain. Stable and instantaneous states. Kolmogorov's forward and backward differential equations. Sufficient conditions for their validity. Conservativity. Pure jump processes. Feller's minimal solution of the differential equations for a given differential matrix. Honest and dishonest solutions. Applications. Linear birth and death processes.

Continuous time Markov branching processes. Their generating functions. Mean population size. Extinction probability. Asymptotic results in the critical, subcritical and supercritical cases.

Renewal processes. The renewal function. Renewal equations and their solutions. The elementary and the "key" renewal theorem. Residual lifetime and age distributions. Modified and equilibrium renewal processes.

Diffusion. Brownian motion and the Wiener process. The diffusion equation and the transition density. The reflection principle. Hitting time and their distributions. The maximum and the zeros of the process.

References

Each of the following general references covers one or more of the topics discussed.

1. D.R. COX and H.D. MILLER: The Theory of Stochastic Processes. (Methuen, 1965). (Recently re-printed in a paperback edition.)
2. S. KARLIN: A First Course in Stochastic Processes. (Academic Press, 1966).
- 2*. S. KARLIN and H.M. TAYLOR: A First Course in Stochastic Processes. 2nd edition (Academic Press, 1975).

Note: 2* is a rewriting and substantial extension of about half the material in 2.

3. G.R. GRIMMETT and D.R. STIRZAKER. Probability and Random Processes. (Oxford University Press, 1982).

MT382Numerical Analysis: Advanced Topics

This course follows on from the second year course MT226, which is the normal pre-requisite. It contains the development of topics in Numerical Analysis, particularly the solution of partial differential equations, and approximation theory. The treatment is analytical, and programming is not required. The course would form a suitable complement to courses in Mathematical Methods (MT320, 360).

Pre-requisites MT226 (MT227 possible).

Combinations May not be taken with MT343 or MT383.

Syllabus1. Approximation Theory

Introduction, examples of problems, existence and uniqueness questions
Chebyshev approximation. Relation between smoothness and rate of
convergence (Jackson's theorem). Characterisation. Least squares
approximation-relation to uniform approximation.

Convergence of interpolation schemes. Spline approximation,
construction of splines. (10)

2. Boundary-value Problems

Finite difference methods. Convergence of difference schemes.
Variational methods. (6)

3. Partial Differential Equations

Finite difference methods.

Elliptic - Poisson's equation on a rectangle. Iterations for linear
systems (Jacobi, Gauss-Seidel, SOR). Finite element approach.

Parabolic Heat equation, explicit and implicit methods, Crank-Nicolson.

Convergence of finite difference scheme. Analysis of stability.

(14)

Text Book

Isaacson, E. and Keller, H.B. "Analysis of Numerical Methods". Wiley.

Also recommended -

Dahlquist, G. and Bjorck, A. "Numerical Methods". Prentice Hall.

Powell, M.J.D., Approximation Theory and Methods. Cambridge University Press.

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MT383 - Numerical Analysis: Differential Equations

This course requires a background of MT226 or MT227 and is a natural continuation of these courses. The overall aim of the course is to introduce some of the basic ideas required in the practical numerical solution of certain types of differential equations. As is known, differential equations arise naturally in the mathematical modelling of many physical problems. In most practical cases these problems cannot be solved by purely analytical techniques and so numerical methods have to be applied - hence the importance of this course. It begins with boundary-value problems in ordinary differential equations and then extends the appropriate ideas to partial differential equations. The work involves the application of basic analysis (for example the essential tool is Taylor's Theorem in one or more variables) and develops further some of the basic ideas of MT226 or MT227. The course complements other third year Numerical Analysis courses and third year applied courses.

MT382 is a barred combination.

Coursework assignments form part of the assignment.

Syllabus

1. Solution of non-linear equations.
2. Ordinary differential equations, two-point boundary value problems. Linear case, superposition of independent solutions, finite difference approximations. Non-linear case, shooting methods. Chebyshev series solutions. (8)
3. Partial differential equations
 - (a) Finite element methods for ordinary differential equations, extension to elliptic equations, direct method of solution. (4)
 - Finite differences for elliptic problems.
 - Iterative solution, as time permits. (5)
 - (b) Parabolic. Explicit and implicit schemes. Stability and convergence. Direct solution of tri-diagonal equations. (5)
 - (c) Hyperbolic. Solution along characteristics. Lax-Wendroff methods for systems of hyperbolic equations. (4)

Text Books:

Most of the material can be found in appropriate parts of:-

- Froberg, C.E. Introduction to Numerical Analysis, Addison-Wesley.
 Schred, F. Numerical Analysis, Schaum.
 Mitchell, A.R. Computational Methods in Partial Differential Equations, Wiley.
 Smith, G.D. Numerical Solution of Partial Differential Equations, Oxford.

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MT385 - Linear Programming and Applications

Linear programming is concerned with problems that can be reduced to: find the n -vector $\underline{x}$ such that $A\underline{x} = \underline{b}$, $\underline{x} \geq \underline{0}$ and $\underline{c}^T \underline{x}$ is minimised; where A , $m \times n$ with $m < n$, $\underline{b}$ and $\underline{c}$ are given. This abstract description misses the enjoyable nature and intrinsic interest of the material. Many "real-life" situations can be put in this form, one example is that of a manufacturer who wishes to determine the best amount of various products to manufacture subject to the constraints of using only the resources at his or her disposal. An intriguing method of solving any problem of this kind, the simplex method, is developed and analysed. In certain cases of these problems, e.g. cheapest transportation schemes, maximum flows in networks, optimum assignment of personnel, the matrix A has a particular structure and special, more compact methods are derived.

The use of the word "programming" in this context pre-dates its use in connection with computers, and this does cause some confusion. There is no direct connection or involvement with computer programming. The course is taught directly from the recommended text-book which provides a complete set of notes and examples.

The course MT386 is concerned with optimisation of non-linear functions with non-linear constraints and together with MT385 is part of the wide field of Operations Research.

The only mathematical techniques needed are essentially results from linear algebra MT112 which is the only prerequisite.

The course may not be taken with MT342, which also covers sections 1, 2, 3.

SyllabusTheory and Simplex Method

1. Introduction, simple examples, convexity, conversion to standard form. Extreme points, basic feasible solutions, degenerate solutions, theorems on nature of minimising solutions. (3)
2. Theory of simplex procedure and simplex tableau. Examples. Description as matrix operations. Artificial basis technique. (4)
3. Duality: definition of symmetric and unsymmetric dual linear programs. Duality theorem. Examples and interpretation of dual programs. Equilibrium theorem. (4)
4. Revised simplex method, and comments on practice. (2)

Special Problems and Applications

5. Parametric Linear Programming and Sensitivity Analysis. (2)
6. Transportation problem, categorised optimal assignment problem. Examples. Duality. (4)
7. Network flows. (3)

8. Simple assignment problem, optimal assignment (marriage) problem.
(4)

9. Game theory.

Text Book

The course text is

"Linear Programming and Applications", W. McLewin, Input Output Publishing Company, September 1980.

MT386Non-linear Optimisation

This course is concerned with the problem of finding the optimum (maximum and minimum) value of a function of many variables, both with and without constraints. The ideas and methods are relatively new, and are of considerable importance since many industrial, economic and sociological problems consist of deciding how to operate a given system most effectively (optimally). The treatment could be regarded as constructive analysis of functions of n -variables and although numerical considerations colour the approach the course does not require second year Numerical Analysis. In a sense the problem is a generalisation of Linear Programming, though the methods are very different: where linear programming uses and gives insight into techniques of elementary linear algebra, nonlinear optimisation uses and brings to life ideas from analysis and geometry in n -space.

There are no barred combinations.

The only prerequisites are MT110/111 and MT112/118. A third year honours level of mathematical sophistication and a flexible and fairly committed approach are the most important requirements. A "pre-course" handout should be available at the end of the Michaelmas term.

Syllabus1. Introduction

Statement of problems; Taylor series in n dimensions; conditions for optimality; concept of direction of search.

2. Unconstrained optimisation

Ad hoc methods: alternating directions and steepest descent. (1)

Methods based on a quadratic model:

- a) without derivatives: conjugate direction methods;
- b) with first derivatives: conjugate gradient methods;
- c) with first and second derivatives: Newton methods. (5)

Quasi-Newton methods: Quasi-Newton equation and derivation of updating formulae, general theory of Quasi-Newton methods, recent extensions. (5)

Details of line search. (1)

Methods without line searches: pattern search and simplex methods. (1)

Sum of squares problems: Gauss-Newton and Levenberg-Marquardt. (2)

3. Constrained Optimisation

Theory of constrained optimisation, first order conditions for a solution. (4)

Linear constraints: direct methods for minimising a general function subject to linear equality constraints; active set strategies for linear inequality constraints. (4)

Non-linear constraints: barrier function methods; penalty function methods; methods based on Lagrangian Functions. (5)

Books

The most suitable book is:

Gill, Murray and Wright, "Practical Optimisation", Academic Press, 1981.

Other useful books are:

Avriel, M., "Nonlinear programming: Analysis and Methods" Prentice-Hall, 1976.

Bazararaa, M.S. and Shetty, C.M., "Nonlinear Programming: Theory and Algorithms", John Wiley, 1979.

Gill, P.E. and Murray, W. (Ed.), "Numerical Methods for Constrained Optimisation" Academic Press, 1974.

Luenberger, D.G., "Introduction to Linear and Non-linear Programming", Addison-Wesley, 1973.

Fletcher, R. "Practical Methods of Optimisations" Vols. I & II, Wiley 1980.

Grieg, M., "Optimisation", Longmans.